Project II

Instructions: Complete one of the following suggested probability projects.

1. Read Example E Bayesian Inference, Rice page 94 - 95.
   Re-work the problem using a $Beta(\alpha, \beta)$ prior instead of the $Unif(0, 1)$ prior used in the example. Use
   \[ f_\theta(\theta) = \frac{\Gamma(\alpha + \beta)}{\Gamma(\alpha)\Gamma(\beta)} \theta^{\alpha-1}(1 - \theta)^{\beta-1} \]  \hspace{1cm} (1)
   for $0 \leq \theta \leq 1$. Determine the posterior density $f_{\theta|X}(\theta|x)$.

2. Read Example C Random Walk, Rice page 140 - 142.
   Write an R program to simulate Brownian Motion for $n = 250$ values. Plot the values. Repeat 10 times. Compare your pictures to the last 250 closing values of a stock traded on the NYSE. Compare your pictures to the last 250 points of a major stock market average such as the DJIA.

3. Read Example B, rice page 154.
   Simulate $n = 200$ values from the $BVN(0, 0, 2, 3, .75)$. Fit the linear model with only the slope coefficient. Compare the the estimate to the $\rho = 0.75$.

4. Monte Carlo Integration.
   Let $U_1, U_2, ..., U_n$ be independent uniform random values from the interval $[a, b]$. These values have density $f(u) = 1/(b - a)$ on that interval. Then
   \[ E[g(U_i)] = \int_a^b g(u) \frac{1}{b - a} \, du \]  \hspace{1cm} (2)
   so the original integral
   \[ \int_a^b g(x) \, dx \]  \hspace{1cm} (3)
   can be approximated by $(b - a)$ times a sample mean of $g(U_i)$.
   Use Monte Carlo integration to estimate the following integrals. Compare with the exact answers, if known.

   (a)
   \[ \int_0^1 x \, dx \]

   (b)
   \[ \int_1^\pi e^x \, dx \]
(c) \[ \int_0^\infty e^{-x} \, dx \]

(d) \[ \int_0^1 \frac{1}{\sqrt{2\pi}} e^{-x^2/2} \, dx \]

(e) \[ \int_0^2 \frac{1}{\sqrt{2\pi}} e^{-x^2/2} \, dx \]

(f) \[ \int_0^3 \frac{1}{\sqrt{2\pi}} e^{-x^2/2} \, dx \]

(g) \[ \int_0^1 \int_0^1 e^{-(x+y)^2} (x+y)^2 \, dx \, dy \]

5. If you have another idea for a project, please propose it during office hours for approval.