If we live with a deep sense of gratitude, our life will be greatly embellished.

Hypothesis Test I

- Logic of hypothesis test
- Rejection region and p-value
- Test for one proportion
Example

- A pharmaceutical company wants to be able to claim that for its newest medication the proportion of patients who experience side effects is less than 20%.

Q. What are the two possible conclusions (hypotheses) here?

The Logic of Hypothesis Test

“Assume Ho is a possible truth until proven false”

Analogical to

“Presumed innocent until proven guilty”

The logic of the US judicial system
Steps in Hypothesis Test

1. Determine the null (Ho) and alternative (Ha) hypotheses
2. Find an appropriate test statistic and pre-set the level of significance (called $\alpha$ level)
3. Assuming Ho is true, find rejection region.
4. Reject Ho if the test statistic falls into the rejection region.
5. Report the result in the context of the situation

Alternative steps for 3 & 4:
3. Assuming Ho is true, find p-value.
4. Reject Ho if p-value $< \alpha$ level.

Determine Ho and Ha

- Ho: nothing is happening (no relationship, no difference, …)
- Ha: something is happening (there is a relationship, there is a difference, …)

- The “=” sign must be in Ho.
- If possible, set what we hope to prove as Ha.
Example

- \( p = \% \) of users who will experience side effects

Q. What are Ho and Ha here?

Example

Logic: Assume Ho is possibly true until proven false.

Data: 17% of 400 patients who have experienced side effects

How likely is \( \hat{p} = 17\% \) if Ho is true (\( p \geq 20\% \))?

If very unlikely \( \Rightarrow \) reject Ho
If not very unlikely \( \Rightarrow \) cannot reject Ho
Rejection Region

How extreme is the observation is too extreme?

- Rejection Region is the region when the test statistic falls in, we will “reject Ho”
- The rejection region is the most extreme region, determined by the $\alpha$ level and the type of Ha

P-Value

- p-value is the probability of seeing as extreme as (or more extreme) what we observe, given Ho is true.
- The smaller p-value is, the less likely that what we observe will occur given Ho is true.
- Smaller p-value means stronger evidence against Ho.
P-Value

- The level of significance (called $\alpha$ level) is usually 0.05

- $p$-value $> \alpha \Rightarrow$ fail to reject $H_0$ (??)
- $p$-value $\leq \alpha \Rightarrow$ reject $H_0$ (= accept $H_a$)

Report the Conclusion

- Reject $H_0$: the data shows strong evidence supporting $H_a$
  Eg. The data shows strong evidence that the proportion of users who will experience side effects is less than 20%.
- Fail to reject $H_0$: the data does not provide sufficient evidence supporting $H_a$
  Eg. Based on the data, there is not sufficient evidence to support the proportion is less than 20%
Testing Hypotheses about a Proportion

- Three possible Ho and Ha

<table>
<thead>
<tr>
<th>Ho</th>
<th>Ha</th>
<th>Type</th>
</tr>
</thead>
<tbody>
<tr>
<td>$p = p_0$</td>
<td>$p = p_0$</td>
<td>Two-sided</td>
</tr>
<tr>
<td>$p &gt; p_0$</td>
<td>$p &lt; p_0$</td>
<td>One-sided (lower-tailed)</td>
</tr>
<tr>
<td>$p &lt; p_0$</td>
<td>$p &gt; p_0$</td>
<td>One-sided (upper-tailed)</td>
</tr>
</tbody>
</table>

Write them all as $p=p_0$ in the future

The z-test for a Proportion

- When 1) the sample is a random sample
  2) $n(p_0)$ and $n(1-p_0)$ are both at least 5,
  an appropriate test statistic for $p$ is

\[ z = \frac{\hat{p} - p_0}{\sqrt{\frac{p_0(1-p_0)}{n}}} \]
Rejection Region

Computing the p-Value for the Z-Test

Upper-tailed test

\[ P\text{-value} = P(Z > z^*) \]
Computing the p-Value for the Z-Test

**Lower-tailed test**

\[
P\text{-value} = P(Z < z^*)
\]

**Two-tailed test**

\[
P\text{-value} = P(|Z| > |z^*|) = 2 \times P(Z > |z^*|)
\]
Example

1. $H_0: p \geq 20\%$ vs. $H_a: p < 20\%$
2. Z-test statistic; $\alpha = 0.05$
3. Find rejection region or p-value
4. Decide if reject $H_0$ or not
5. Report the conclusion in the context of the situation