A Discounting Model for Decisions With Delayed Positive or Negative Outcomes

Mary Kay Stevenson
Purdue University

Many decisions involve future consequences. Although research in incentive theory, delay of gratification, and impulse control has demonstrated preferences for more immediate consequences, very little research has been done on the discounting process underlying the evaluation of delayed consequences. The present studies were designed to assess the subjective values of investments and credit plans that were defined as temporally remote outcomes. Three studies used investment stimuli that varied in the amount of interest to be gained in the future. The fourth study used credit plans and investments that varied in the amount of money to be lost in the future. Converging support was obtained for a ratio discounting function for time. The results are discussed in relation to (a) the evidence for a ratio discounting function for time, (b) the presence of task-dependent response patterning, and (c) the comparison of subjective values for money and time derived with risky and riskless stimuli.

Most of the empirical work on the effect of delayed reward has been done in psychology. Basic learning theory has focused on three manipulations in defining the incentive value of contingent outcomes: magnitude, probability, and immediacy. The most effective incentive conditions are high in magnitude and certain to occur immediately after the response. Magnitude, probability, and delay have been used to describe the relative impact of both positive and aversive consequences. In general, delaying the consequence of a response reduces its impact on behavior (Renner, 1964; Tarpy & Sawabini, 1974).

Some of the human research in delayed consequences has been generated from self-control theory. It began with a search for individual, cultural, and situational correlates of the tendency to choose a larger reward that is delayed over a smaller reward that is immediately available (e.g., Mischel & Metzner, 1962). Then studies were designed to determine which situational variables would enhance the tendency of subjects to choose the delayed reward (e.g., Miller & Karmiol, 1976; Mischel & Ebbeson, 1970; Moore, Mischel, & Zeiss, 1976; Yates & Mischel, 1979).

Several interpretations of the effect of delay on choice responses are feasible. For example, Mischel and Grusec (1967) applied Rotter's (1954) theory of expectancy value to delayed consequences. According to this view, delayed consequences are associated with an implicit risk value; therefore delayed positive consequences are avoided because they are less certain. Delay could reduce the affective value or the salience of a consequence (Mischel & Grusec, 1967). These interpretations have quite different implications for understanding the incentive value of future consequences. The analysis of the cognitive processes underlying the evaluation of temporally remote consequences requires specifying how time is combined with magnitude in determining the desirability of a consequence. The current research is focused on this discounting process.

When a positive outcome is described as temporally remote, its value is reduced. Two simple operations may be used to describe the change in value that occurs as a function of temporal proximity. A subtractive discounting operation indicates that the

I would like to thank Michael H. Birnbaum and Douglas Medin for their valuable comments on the first draft of this article. Appreciation is also extended to Roberta Schlesmann for her assistance in collecting the data and in the preliminary analyses and to Marilyn Douglas for assisting in the data collection.

This research was supported in part by the Army Research Institute under Contract MDA903-85-K-0366.

Correspondence concerning this article should be addressed to Mary K. Stevenson, Department of Psychological Sciences, Purdue University, West Lafayette, Indiana 47907.
The effect of time on the evaluation of an outcome is independent of its value. According to the subtractive operation, the value of an outcome is reduced by a constant amount. The ratio discounting operation indicates that the effect of time on the evaluation of an outcome is relative to its value. Subjects using a ratio discounting function reduce the value of an outcome proportionately. If waiting is an aversive component of an outcome, either a subtractive or a ratio operation could be used to describe the effect of time. If future outcomes are viewed as implicitly risky (i.e., time introduces an uncertainty component) then the ratio operation would be more likely to describe the discounting function because events that are defined with explicit probabilities are reduced in proportion to their values (Anderson & Shanteau, 1970; Lynch, 1979; Shanteau, 1975; Tversky, 1967b).

Örlandahl and Sjöberg (1979) reported a series of studies using human subjects that were designed to test both the subtractive and ratio discounting functions for delayed positive outcomes. They used several types of response measures and assumed the response functions were linearly related to the models tested, although magnitude estimations and graphic ratings were reported to be nonlinearly related to each other. They concluded that in general it was difficult to eliminate either the ratio or subtractive discounting model: the ratio model was more accurate in describing the magnitude estimates, and the subtractive model was more accurate in describing the graphic ratings. When different tasks or response measures require alternative models, the results could indicate different discounting strategies or a nonlinear relation between the observed responses and the modeled values in one or both measures. In order to determine which of these components are affected, the experimental design must provide a means of defining the appropriate response function.

The first three studies to be described used investment stimuli and two types of rating tasks to determine how future positive consequences are discounted. Investments were described in terms of the amount of money to be earned, the length of time required to obtain a return, and, for one task, by the degree of risk involved. The fourth experiment used negative outcomes. Preference for credit plans and investments that varied in the amount of money that could be lost were assessed. The utility of these stimuli were measured in order to determine whether the values of the outcomes were discounted in proportion to the subjective magnitudes of the return or whether the discounting process is independent of the task or type of response measure used to quantify the future utility. Therefore, converging evidence across experimental tasks is assumed to be important in establishing the validity of the discounting model.

**Experiment 1**

The purpose of this experiment was to determine how subjects would evaluate risky investments with an explicit time delay. Each investment was described by the amount of the return, the
length of time to reach maturity, and the probability of obtaining a return (i.e., risk). An example trial is shown in Figure 1 (bottom panel). Subjects rated the desirability of these investments on a unipolar graphic rating scale. A general model for this evaluation process is shown in Figure 1 (top panel). The three attributes, probability \((p)\), magnitude \((m)\), and time delay \((t)\) are transformed to their subjective scale values \((S_p, S_m, S_t)\), respectively) according to the subject’s perspective. This transformation is assumed to depend on the physical values of the single attributes. These subjective values are then combined to produce the overall impression of the investment \((\psi_{mpt})\). Because previous research has supported the multiplicative integration function for magnitude and probability (e.g., Anderson & Shanteau, 1970; Lynch, 1979; Shanteau, 1975; Tversky, 1967b), the current analyses assume that this operation is multiplicative. Finally, because previous research (Anderson, 1982; Birnbaum, 1982; Ördendahl & Sjöberg, 1979) has indicated that different response scales are nonlinearly related, a response function is used to map the implicit ratings \((\psi_{mpt})\) onto the observed response measure \((R_{mpt})\). Presumably this response function represents the subject’s ability to map or scale the subjective values of the stimuli on the rating scale defined by the task. The following models describe a multiplicative combination function for magnitude and probability and both the subtractive and ratio discounting function for the time factor:

\[
R_{mpt} = J_m(S_p S_m S_t) + e_{mpt}, \quad (1a; \text{multiplicative})
\]

\[
R_{mpt} = J_d(S_p S_m - S_t) + e_{mpt}, \quad (1b; \text{dual distributive})
\]

\[
R_{mpt} = J_d[S_p S_m - S_t] + e_{mpt}, \quad (1c; \text{distributive})
\]

These models can be discriminated on the basis of the rank ordering of the predictions. A ratio discounting function for time indicates that the effect of time depends on the magnitude of the outcome. If subjects evaluated the return on the investment by putting it in proportion to the amount gained over a fixed period of time, a ratio discounting function would represent this strategy. A subtractive discounting function for time indicates that the effect of time is independent of the magnitude of the outcome. In this case, time would be viewed as an aversive component that detracts from the value of the return. This experiment was designed to test the discounting function for time in the context of risk while assuming a multiplicative relation between magnitude and probability so as to concur with previous research.

**Method**

**Task.** Each subject was instructed to act as an investment counselor who had a client wishing to invest $1,000. They were asked to evaluate each investment and to indicate on the continuous rating scale the desirability of the investment conditions (see Figure 1, bottom panel). The “worst” and “best” investment in the stimulus set served as anchors for the response scale and were identified at the left and right endpoints, respectively.

**Design.** The 80 trials were constructed from a factorial design of four amounts ($40, $80, $160, $320), four time delays (3 months, 6 months, 1 year, 3 years), and five probabilities (.1, .3, .5, .8, 1.0).

**Procedure.** Each subject was instructed individually and several practice trials were used to check the subject’s understanding. Then all 80 investment stimuli were presented. A different pseudorandom sequence of the 80 stimuli was used for each subject and each session. Sequences were randomly generated with the constraint that similar investments be separated by a few trials. Each subject rated all of the stimuli twice in two sessions scheduled 1 week apart.

**Subjects.** Twenty undergraduate students (6 males and 14 females) from an introductory psychology course volunteered to participate for course credit.

**Results**

The mean ratings obtained for each investment, averaged across sessions and across subjects are shown in Figure 2 as points. The lines represent predictions to be described later. The means are plotted according to the attributes of the investment: magnitude of the gain, time to maturity, and probability of success. Although the trend in the means appears parallel for the combination of magnitude and risk, the results of an analysis of variance (ANOVA) indicated a significant interaction between probability and magnitude, \(F(12, 228) = 10.39, p < .05\), and probability and time, \(F(12, 228) = 4.15, p < .05\).

These results support the initial assumption about the multiplicative combination process when a linear response function is assumed. Because there was no significant interaction between time and magnitude, \(F(9, 171) = .75, p > .05\), the results obtained assuming a linear response model indicate that the distributive model (Equation 1c) best describes the way probability, magnitude, and time are combined to produce the overall rating.

The following analyses were completed in order to compare the ordinal consistency of the multiplicative, distributive, and dual distributive models (Equations 1a–1c) with the observed responses while assuming that the response function may be nonlinearly related to the implicit ratings \((\psi_{mpt})\). Similar response patterns were obtained for the individual subjects and across sessions so the mean ratings across sessions were averaged across subjects for the following analyses. The response function was parameterized using the integrated spline approach (Winsberg & Ramsay, 1981). In general, spline functions are piecewise polynomial functions of a given degree (deBoor, 1978; Winsberg & Ramsay, 1981). The segments of the function are defined by a particular knot sequence that is used to increase the flexibility of the polynomial function. The number of parameters that describe the form of the nonlinear function are determined by the number of knots selected. The order of the spline function \((K)\) defines the degree of the polynomial \((K - 1)\) for the function. The predicted values for each model \((\psi_{mpt})\) were mapped onto the second-order spline function having four segments to define the form of the function. Each model was tested using the same number of parameters and order for the spline functions although the placement of the knots varied according to the form of the function. For each model six estimated parameters for the response function (including an intercept), four scale values for magnitude, four scale values for time, three scale values for probability (the lowest and highest probability scale values were fixed at .1 and 1.0, respectively) were estimated using Marquardt’s compromise procedure (Draper & Smith, 1981). A plot of the predicted and observed values for each of the models is shown in Figure 3 (top, middle, and bottom panels) with the corresponding response function parameters and the vertical lines marking the knot locations. Each group shows the predicted and observed values plotted as a function of the “value” of the investment derived from the model. The points represent...
the observed values for each investment. The percentage of variance in the ratings accounted for by the dual distributive model (96.90%) and the distributive model (97.73%) were high, but the multiplicative model was the highest (99.07%). Note that relatively large deviations from the predicted values were evident for the dual distributive and distributive models that were not evident for the multiplicative model. Furthermore, these discrepancies occurred because of the ordinal differences between the predicted and observed values and therefore could not be eliminated by increasing the flexibility of the response function.

A comparison $F$ statistic was computed as an approximate goodness of fit test (Draper & Smith, 1981). The sum of the squared deviations of the modeled predictions from the mean ratings represented an estimate of the lack of fit. In order to evaluate the extent of these deviations, the pure error was estimated by computing the sum of squared deviations in the means across session replicates. The comparison $F$ statistic (i.e., the ratio of the mean square lack of fit to the mean square of the pure error estimate) indicated no significant difference between the observed values and those generated by the transformed multiplicative model, $F(63, 80) = .71, p > .05$. The results obtained for the distributive, $F(63, 80) = 1.72, p < .05$, and the dual distributive, $F(63, 80) = 2.34, p < .05$, models indicated significant deviations in the fit of these models. Therefore, the multiplicative model (Equation 1a) was considered the most appropriate representation for the rating of risky investments.

**Discussion**

Support for the multiplicative model indicates that a ratio discounting function describes the effect of time on the value of interest to be gained in the future. When subjects were asked to evaluate the worth of investments that varied in the magnitude of the return, time delay, and the probability of a gain, the effect of the time variable depended on the magnitude of the outcome. These results contradict the conclusion drawn by Örtendahl and Sjöberg (1979), who reported that a subtractive combination rule was more accurate in describing the effect of time on the value of an outcome using graphic ratings of bets. However, they assumed a linear response function in their analysis. In agreement with the current findings, they obtained support for a ratio combination rule using magnitude estimates. Because they assumed a linear response function in their analyses of the graphic ratings, the apparent contradiction may be due to the nonlinear relation between graphic ratings and magnitude estimates that they reported. Consistency in the discounting function across response measures that are nonlinearly related can be obtained only if the nonlinearity in the response scale is defined in the theoretical framework and model. In order to test the hypothesis that different types of tasks or response measures can be used to obtain converging evidence for a discounting function if the form of the response function is allowed to vary across tasks, a different design and task were used in the next experiment.
Experiment 2

The purpose of this experiment was to determine whether investment stimuli would be compared by a discounting preference process or by an attribute comparison strategy. Subjects were asked to compare investment stimuli and rate their degree of preference for one investment with respect to the other. An example stimulus is shown in Figure 4 (bottom panel). Each investment set (labeled A and B) was defined by a factorial combination of several monetary values and time requirements. Subjects indicated how much they preferred one investment to the other by marking a bipolar continuous rating scale with the center defined as indifference.

Discounting Preference Model

With the discounting preference model, it is assumed that subjects evaluate each investment first by discounting the amount of interest to be gained by the length of time required to reach maturity. Then a preference rating is generated by comparing the subjective values of the two investments. This model is described in Figure 4 (top panel). The physical values of the magnitude of the return ($\phi_m$, $\phi_b$) and the time requirement ($\phi_t$, $\phi_d$) are transformed into subjective reference values representing the respondent's point of view. This process produces scale values for both the magnitude ($S_m$, $S_b$) and the time attributes ($S_t$, $S_d$). This scaling process is assumed to depend only on the physical values of the single attributes. The investments are then evaluated individually by discounting the amount to be gained by the length of time required by the conditions of the investment. This function is the discounting operation.

Support for a ratio discounting function,

$$\psi = S_m/S_t,$$

(2)

would indicate that temporally remote consequences are evaluated in proportion to the magnitude of the return. In this case, the return of the investment is evaluated as a function of the time required, whereas support for a subtractive model,

$$\psi = S_m - S_t,$$

(3)

would indicate that the discounting process is independent of the magnitude of the return. In this case, the effect of time would be to add the same aversive component to the overall utility of the investment regardless of the magnitude of the future outcome.

The second operation in the discounting preference model describes the comparison of the two investments or the generation of the implicit preference rating. This function describes the process used to compare the two investments and defines the implicit preference rating ($p_{AB}$). Previous research has indicated that the comparison of two stimuli in a preference task can often be represented as a subtractive or differencing process (e.g., Birnbaum, 1974; Winsberg & Ramsay, 1981); therefore this preference operation will be assumed to be subtractive.

Because previous research has indicated that rating tasks using different response scales may produce response measures that are nonlinearly related for the same stimuli (Anderson, 1982; Birnbaum, 1982; Östendahl & Sjöberg, 1979), the present analyses assumes that the observed responses ($P_{AB}$) are monotonically but nonlinearly ($J_p$) related to the implicit preference ($p_{AB}$) as indicated in Equation 4.

$$P_{AB} = J_p(p_A - p_B) + e_{AB}.$$

(4)

This assumption is based on the notion that different types of rating scales produce different types of response patterns or mapping cues. In other words, if the same stimuli were presented to the subject but different rating scales were provided, the pattern
of responses obtained should be monotonically related but not necessarily linear in form. Allowing the response function to be nonlinear leads to some problems in identifying the discounting function.

Two operation models have been proposed as a method of enriching the data base in order to constrain the number of mathematical models that would be appropriate in describing the data (Birnbaum, 1974). For example, a two-operation model may have an additive and a multiplicative component. In a simple factorial design, combining magnitude and time, both the ratio and the subtractive discounting model can describe the same set of data by altering the form of a nonlinear monotonic response function (Anderson, 1970; Birnbaum & Veil, 1974). If a two-operation model represents the cognitive process underlying the task, ordinal constraints distinguish among some of the alternatives for one operation as long as the other operation is known.

If the two operations are identical, there is no ordinal constraint to define the most appropriate model because the monotonic transformation can convert the pattern of one process (e.g., additive) to another (e.g., multiplicative).

By fitting the preference operation to define the scale values for the investments and the form of the response function, the discounting process can be evaluated without the ambiguity of a possible transformation problem. The scale value estimates obtained from this model for each set of investments (A and B) are plotted as a function of the attributes defining them (i.e., magnitude and time). The pattern of these scale values defines the discounting function. A bilinear fan pattern indicates that the discounting function is proportional to the magnitude of the gain (i.e., a ratio model would be supported). A set of parallel lines indicates that the discounting process is independent of the magnitude of the return (i.e., this evaluation process may be

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**Figure 4.** The discounting preference model (top panel) and an example of the investment stimulus (bottom panel). (Each investment was described in relation to the amount to be gained [\(m\)] and the length of time required to reach maturity [\(t\)]. 1. The scaling functions are \(S_m = H_m(\phi_m)\) and \(S_t = H_t(\phi_t)\). 2. The discounting functions are \(\psi_A = I[S_{ma}, S_{ta}]\) and \(\psi_B = I[S_{mb}, S_{tb}]\). 3. The response function is defined as \(P_{AB} = I[\phi_{ma} + \epsilon_{AB}]\).
TEMPORAL DISCOUNTING OF OUTCOMES

The attribute comparison model. (1) The scaling functions are \( S_m = H[\phi_m] \) and \( S_t = H[\phi_t] \). 2. The comparison functions are \( \theta_m = I[S_m, S_a] \) and \( \theta_t = I[S_t, S_b] \). 3. The combination function is \( \rho_m = C(\theta_m, \theta_t) \). 4. The response function is \( P_{mt} = J^\theta_m(\rho_m) + e_{mt} \).

\[ \theta_m = S_{mA} - S_{mB}. \] (6)

If the attributes are compared by evaluating the "difference" in subjective values, this model corresponds to Tversky's (1969) additive difference model of stimulus comparisons. A comparison process also occurs for the time required to obtain a return \( (S_{aA} vs. S_{aB}) \) that produces a scale value \( \theta_t \) for the relative value or the difference in the time requirements.

The preference for Investment A or B is defined by a combination of the scaled attribute comparisons. If all of the attributes defining a situation are positive, a simple summation operation is assumed to define the implicit preference rating \( (\rho_m) \). Because time delay is viewed as a negative factor, it is assumed to detract from the gain attribute. For example, if Investment A produces a larger return than Investment B, but also requires a long time delay, the time factor may outweigh the monetary value and produce a preference for Investment B. Equation 7 summarizes the combination process.

\[ P_{mt} = J^\theta_m(\theta_m - \theta_t) + e_{mt}. \] (7)

(\( J^\theta_m \) is a nonlinear monotonic function that maps the implicit preference ratings onto the observed response scale.)

The analysis of this model is similar to the process described for the discounting preference model. Fixing the combination operation for the attributes and optimizing the fit defines the comparison scale values for the attributes across the stimulus pairs and the form of the response function. These scale values can then be plotted as a function of the two levels of magnitude (or time) defining the investment pairs in order to determine the form of the comparison operation within attributes. If, however, the subtractive model describes both the combination and comparison operations, the lack of ordinal constraints in the system...
will prevent the identification of the operations. It will be possible to transform the equations from subtractive to an equivalent form because there is an explicit assumption that the preference ratings are monotonically related to the implicit preference values. If two operations are needed to describe the comparison and combination processes then the system will be constrained ordinally and it will be possible to eliminate classes of models that would be inconsistent with the ranked order of the preference ratings.

**Method**

**Task.** Subjects rated their degrees of preference for one of the two investments presented on each trial. An example of the investment stimuli and preference scale are shown in Figure 4 (bottom panel). Each subject was instructed to act as an investment counselor who had a client wishing to invest $1,000. They were asked to compare each pair of investments and to indicate their degrees of preference for one of the investments by marking a location on the continuous response scale (see Figure 4, bottom panel). Subjects indicated their degrees of preference for one alternative over the other by responding closer to the appropriate endpoint. The center of the rating scale represented indifference between the two investments. The "best" investment (yielding the greatest return with the shortest delay) and the "worst" investment (yielding the smallest gain with the longest delay) were provided as anchors for these judgments. These anchor investments were explicitly provided so that the subjects knew the range of investment deals that would be presented.

**Design.** The 16 levels or investments for Stimulus Set A were constructed by a factorial combination of four amounts ($40, $80, $160, $320) and four time delays (3 months, 6 months, 1 year, 3 years). The six levels or investments for Stimulus Set B were constructed from a factorial combination of two amounts ($60, $240) and three investment periods (5 months, 9 months, 2 years). Each of the 16 investments of Stimulus Set A was combined with each of the six investments of Stimulus Set B yielding 96 (16 x 6) preference trials.

**Procedure.** Each subject was instructed individually and several practice trials were used to check the subject's understanding of the task. Then each of the 96 preference trials was presented in the format shown in Figure 4 (bottom panel). Subjects rated the 96 investment comparisons twice in two sessions scheduled 1 week apart. A different pseudorandom sequence of 96 trials was used for each subject and each session. Each sequence was randomly ordered with the restriction that similar investments be separated by a few trials. After subjects had completed both sessions, they were asked to identify the best and worst investments that were used as anchors and to describe the decision strategy that they had used.

**Subjects.** Thirty-six undergraduates from an introductory psychology course volunteered to participate for course credit.1 Sixteen males and 14 females participated in two sessions (scheduled 1 week apart) to provide a complete replication of the preference ratings.

**Results**

**Preference model.** The mean preference ratings obtained from each subject for two sessions were averaged across subjects and sessions and are shown in Figure 6 as points. The predictions represented by the lines will be described later. The investments from Stimulus Set A were ordered according to the marginal means of the preference ratings and are given on the abscissa. Each line represents a different investment from Stimulus Set B. The preference model defined in Equation 4 specifies that the relation between Investment Sets A and B should be described by parallel lines. Therefore, the somewhat barrel-shaped pattern shown in Figure 6 indicates that either the difference model is inappropriate or the response function relating the implicit comparisons \( \rho_{AB} \) to the observed responses \( P_{AB} \) is nonlinear. In order to test the adequacy of the difference model, an integrated second-order spline function was used to parameterize the response function \( J_\rho \) as described in Experiment 1. Because the pattern of responses obtained from individual subjects were consistent with the means, the following analyses were performed on the means.

The predicted values for the discounting preference model \( (\psi - \psi) \) were mapped onto a second-order spline function having five segments to define the form of the function. Therefore, the model for the comparison of investments included seven estimated parameters for the response function \( (\psi_j, j = 1 \text{ to } 6 \text{ and an intercept } \psi_0) \), 14 scale values for Stimulus Set A (the lowest and highest scale values were fixed to define the scale unit), and six scale values for Stimulus Set B. These values were estimated to minimize the sum of the 96 data-model squared discrepancies using Marquardt's compromise procedure (Draper & Smith, 1981).

The results of the analysis indicated that the \( J \) function maximizing the fit of the difference model is approximately S-shaped \( (a_i = .36, .16, 2.98, .39, .32, 0) \). The form of this function is shown in Figure 7 (top panel) with the vertical lines indicating the placement of the knot segments. Note that the change in parameter values (increasing vs. decreasing) reflects the direction of acceleration between intervals. Predicted values are repre-

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1 One subject who failed to return for the second session and five subjects who restricted their responses to the endpoints of the scale were dropped from the analyses.
sent by the solid line, and the observed values are plotted as points. The discounting preference model accounts for 99.15% of the variance in preference ratings. The predicted preference ratings are plotted in Figure 6 as lines with the observed values represented by points. The predicted values correspond closely with the observed response pattern in the group means. There do not appear to be any systematic deviations in the predicted values.

The comparison $F$ statistic described in Experiment 1 was also computed to evaluate the fit of the discounting preference model. The comparison $F$ statistic (i.e., the ratio of the mean square lack of fit to the mean square of the pure error) indicated that the mean squared model-data discrepancy was about 2.66 times as large as the mean squared deviation between sessions.

This ratio may be approximately $F$ distributed with (69, 96) degrees of freedom. This result indicates significant deviations ($p < .05$) in the fit of the difference model to the observed preference ratings, but there appear to be no meaningful patterns in the deviations.

Attribute comparison model. A model assuming an attribute by attribute comparison process was also tested to determine whether the present set of data could be better represented with this type of cognitive process. This model was defined in Equation 7. In this model, $\theta_m$ represents the comparison of the investment magnitudes and $\theta_t$ represents the comparison of investment time delays. According to this model, the preference ratings are determined by an additive combination of the attribute values. Because time delay is assumed to have a negative effect on preference, a subtractive process is appropriate for this application. $J^*_p$ is assumed to be a monotonic nonlinear transformation of the attribute composite that will be parameterized to maximize the fit of the attribute comparison model.

The matrix of preference means was reorganized to include eight combinations of magnitudes across investments ($\theta_m$) and 12 time combinations ($\theta_t$). In this analysis 25 parameters were estimated to test the attribute comparison model: 6 scale values for the magnitude attribute (two scale values were fixed), 12 scale values for the time attribute, and 7 parameters for the nonlinear function ($J^*_p$) including an intercept.

The predicted and observed data values are shown in Figure 7 (bottom panel). The attribute comparison model accounted for 96.64% of the variance in preference ratings. In comparison with the deviations shown in Figure 7 (top panel) for the discounting preference model, a wider range of discrepancies was obtained using a model that assumes comparisons were made across attributes. Note that increasing the flexibility of the nonlinear response function would not improve the fit of this model. According to these results, the initial assumption that magnitude was discounted by time for each investment stimulus appears to be more appropriate than assuming an attribute by attribute comparison process. This distinction can be based on ordinal differences in the predicted values derived from the competing models. Therefore, for these stimuli, it appears that subjects compared the subjective values of the investments rather than the individual attributes that varied across the preference alternatives.

Discounting function for magnitude and time. The estimated scale values obtained from the discounting preference model (Equation 4) for Investment Set A and B are plotted in Figure 8 as a function of the four levels of magnitude with a different line for each time delay. The scale values for each magnitude and time delay are the marginal means of the investment scale values after a linear transformation that will be described later. The diverging bilinear fan pattern indicates that a ratio discounting operation describes the effect of time on the utility of investment returns. For example, the ratio discounting function would represent a strategy of defining the amount to be gained in each investment per time unit in order to compare the respective values. With this strategy a $320 gain in 4 years is equivalent to an $80 gain in 1 year assuming that the psychophysical functions for time and magnitude are both identity functions. The same diverging pattern was obtained for both sets of investments.

Scale value estimates. Scale values were obtained for interest rates and times from a rating task in Experiment 1 that defined
the investments as risky (i.e., an explicit probability for the return was included) and from a preference task that paired investments that guaranteed a return (i.e., riskless) in Experiment 2. The marginal means of the investment scale values representing each magnitude and time obtained from the preference discounting model in Experiment 2 were linearly transformed to match the minimum and maximum estimated scale values obtained from the multiplicative model of risky investments in Experiment 1. The psychophysical functions for the stimuli used in both experiments are shown in Figure 9. The scale values obtained for the magnitude of the return of risky investments exceeded the scale values for interest estimated from the riskless investments. The psychophysical functions obtained for the time scale (H) differed across tasks. The scale values (S) for the time values are linearly related to the physical values (φ) in the rating task but are a negatively accelerated function of the physical values for the preference task. The subjective time scale from the riskless investment task exceeded the subjective time scale from the risky investment task.

Discussion

In summary, the results of Experiment 2 are consistent with the hypothesis that subjects were comparing the investments after discounting the magnitude of the return using a ratio discounting function for time. The pattern of scale values obtained for both sets of investments supported the hypothesis that the temporal discounting function was ratio.

Because two operations were required to describe the pattern of preference ratings obtained,

\[ P_{AB} = J_\mu[(S_{mA}/S_{mB}) - (S_{mA}/S_{mB})], \] (8)

the rank order of the stimuli defined by this model is incompatible with several other classes of models that could have been used. For example, Equation 9 represents a difference discounting preference model that is ordinally inconsistent with the ratio discounting preference model:

\[ P_{AB} = J_\mu[(S_{mA} - S_{mB}) - (S_{mA} - S_{mB})]. \] (9)

Equation 10 is derived from Equation 9:

\[ P_{AB} = J_\mu[(S_{mA} - S_{mB}) - (S_{mA} - S_{mB})]. \] (10)

This model describes an attribute comparison process and more specifically represents Tversky's (1969) additive difference model. Russo and Dosher (1983), in reviewing the use of holistic and dimensional judgment strategies, concluded that the holistic evaluation of stimuli is more compatible with a multiplicative combination rule than a dimensional (or attribute) comparison strategy. Because Equation 10 is ordinally identical to Equation 9 it has also been eliminated from further consideration in the evaluation of investment stimuli.

Models 11 and 12 have also been rejected because they are simply nonlinear transformations of Equations 9 and 10, respectively.

\[ P_{AB} = J_\mu[(S^*_{mA}/S^*_{mB})/(S^*_{mA}/S^*_{mB})]. \] (11)

\[ P_{AB} = J_\mu[(S^*_{mA}/S^*_{mB})/(S^*_{mA}/S^*_{mB})]. \] (12)

Therefore, using the constraints inherent in two-operation processes, classes of models that are ordinally incompatible with the discounting preference model can be differentiated under the assumption that the response function translating the implicit preference value to the observed response scale is monotonic.

The response function for the bipolar preference scale in Experiment 2 was approximately S-shaped. This form is similar to those obtained by Winsberg and Ramsay (1981) for preference ratings and by Rose and Birnbaum (1975) for bipolar ratings of quite different types of stimuli. This similarity indicates that the type of task used may determine the patterns of observed ratings. The response function for the unipolar rating scale of Experiment 1 was less complex (i.e., more consistently negatively accelerated across segments). Bipolar preference scales and unipolar rating scales may produce different response patterns. On a bipolar rating scale, the center of the scale may serve as the 0 reference point (i.e., the point of indifference). Small differences near the reference point may be exaggerated (Rose & Birnbaum, 1975). Or, as the distance from the reference point increases, subjects may be more likely to underestimate the differences between stimuli on the rating scale. With unipolar rating scales one end-point (i.e., the worst or best stimulus) may be used as a reference point for the responses. For unipolar rating scales, the exaggeration of small differences (or the underestimation of the differ-

Figure 8. The scale values derived from the discounting preference model for Investments A and B plotted as a function of the scale values ($S_m$) of the magnitude of the return. (Each line corresponds to a different time requirement represented in the investments.)
The psychophysical functions obtained for time also varied across tasks. For risky investments, there was a nearly linear correspondence between the physical and subjective values for time. For riskless investments, the subjective values were a negatively accelerated function of the physical values for time. In other words, the subjective difference between 3 months and 6 months was greater than the subjective difference between 2 years and 3 years. Variability in the form of the scaling functions may be due to the presence and absence of explicit probabilities (i.e., risk) in the investments. The shift in scale values may involve the notion that time delay represents an implicit risk (Mischel & Grusec, 1967). An alternative interpretation of the shift in scale values may relate to the degree of importance the time delay had in rating the investment relative to the probability of a return. Differences could also be due to the fact that different subjects were tested in the two task conditions. Because the subjective values of a stimulus set may vary across individuals, the same group of subjects should be measured on both tasks. If response functions vary across tasks, establishing the agreement of the scale values across those conditions would lend stronger support to the interpretation of the discounting function by imposing additional constraint on the solution.

Experiment 3

Experiment 3 was designed to determine whether the variability in the subjective scales for interest and time across tasks was due to the task or to the use of different subjects across task...
conditions. According to expected utility theory (Schoemaker, 1982) valuation scales derived in the context of riskless outcomes are expected to be only monotonically related to subjective scales derived from risky outcomes. Therefore the lack of scale convergence obtained across tasks in Experiment 1 and 2 could be due to the use of risky investments in Experiment 1 and riskless investments in Experiment 2.

In order to differentiate between these two explanations, the same subjects were asked to evaluate the desirability of risky investments identical to those used in Experiment 1 and to indicate their degree of preference for riskless investment stimuli identical to those used in Experiment 2. The current design also provides an opportunity to test the consistency of the response functions derived for each type of response measure across subjects and presentation methods. All the stimuli were presented on a video screen. Finally, the validity of the temporal discounting model can be tested across subjects. The multiplicative, dual distributive, and distributive models (Equations 1a–1c, respectively) were tested for the risky investment task. Both the discounting preference model (Equation 8) and the attribute comparison model (Equation 7) were evaluated for the investment preference task.

### Method

**Task.** The rating task was identical to that of Experiment 1, and the preference task was described in Experiment 2.

**Design.** In order to limit the time required to complete both tasks to approximately 1 hour, the number of stimuli used in each task was reduced.

For the risky investment rating test, 48 trials were constructed from a factorial design of three levels of interest ($60, $160, $320), four time delays (3 months, 6 months, 1 year, 3 years), and four levels of probability (.1, .5, .8, 1.0).

For the preference task, 12 levels or investments for Stimulus Set A were constructed by a factorial combination of three levels of amount ($100, $240) and two investment periods (9 months, 2 years). Each of the 12 investments of Stimulus Set A was combined with each of the four investments of Stimulus Set B yielding 48 (12 × 4) preference trials.

Procedure. As in the first two studies, each subject was instructed individually and several practice trials were used to check the subject’s understanding of each task. Half of the subjects worked on the preference task first, the other half completed the rating task first. Unlike the first two experiments, each trial was presented on a video terminal. Each of the risky investment rating trials was presented in the format shown in Figure 1 (bottom panel). Each of the preference trials appeared on the screen in the same format shown in Figure 4 (bottom panel). The cursor was positioned in the center of both the preference scale and the rating scale. Subjects moved the cursor to a position that represented their response and pressed “/”. A mark appeared on the scale shown on the screen. If the subject was satisfied with this location, a second input response recorded the location of the mark and initiated the next trial. Otherwise, the subject was free to move to a new location and repeat the response procedure.

Subjects completed both tasks in each session. Two sessions scheduled 1 week apart provided a complete replication to check for reliability. A different pseudorandom sequence of trials was used for each subject, session, and task. After the subjects had completed both sessions, they were asked to recall the best and worst investments that were used as anchors and to describe the decision strategy that they had used for each task.

**Subjects.** Twenty-one undergraduates from an introductory psychology course volunteered to participate for course credit.

### Results

**Risky investment ratings.** The mean ratings obtained for each investment averaged across sessions and subjects are shown in

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Table 1

<table>
<thead>
<tr>
<th>Probability</th>
<th>.1</th>
<th>.5</th>
<th>.8</th>
<th>1.0</th>
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<tr>
<td>Gain</td>
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<td></td>
<td></td>
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</tr>
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<td>3 months</td>
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<td>15.12</td>
<td>28.19</td>
<td>39.21</td>
<td>44.30</td>
</tr>
<tr>
<td>O</td>
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<td>28.84</td>
<td>40.14</td>
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<tr>
<td>$160</td>
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<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>P</td>
<td>20.38</td>
<td>35.30</td>
<td>45.76</td>
<td>52.70</td>
</tr>
<tr>
<td>O</td>
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<td>45.66</td>
<td>51.76</td>
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<td>$320</td>
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<td>40.11</td>
<td>51.71</td>
<td>59.67</td>
</tr>
<tr>
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<td>39.53</td>
<td>51.48</td>
<td>59.76</td>
</tr>
<tr>
<td>6 months</td>
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</tr>
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</tr>
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<td>48.21</td>
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</tr>
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</tr>
<tr>
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<td>19.99</td>
<td>34.81</td>
<td>45.26</td>
<td>52.08</td>
</tr>
<tr>
<td>O</td>
<td>18.98</td>
<td>34.13</td>
<td>45.44</td>
<td>52.02</td>
</tr>
<tr>
<td>3 years</td>
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</tr>
<tr>
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<td></td>
<td></td>
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</tr>
<tr>
<td>P</td>
<td>1.39</td>
<td>17.76</td>
<td>26.72</td>
<td>32.10</td>
</tr>
<tr>
<td>O</td>
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<td>24.82</td>
<td>31.64</td>
</tr>
<tr>
<td>$160</td>
<td></td>
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</tr>
<tr>
<td>P</td>
<td>9.79</td>
<td>23.53</td>
<td>33.55</td>
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<tr>
<td>O</td>
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<tr>
<td>$320</td>
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</tr>
<tr>
<td>P</td>
<td>14.71</td>
<td>27.62</td>
<td>38.62</td>
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<tr>
<td>O</td>
<td>14.56</td>
<td>28.79</td>
<td>37.08</td>
<td>45.74</td>
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</table>
The patterns of the means replicate the trends observed in Experiment 1. The results of an ANOVA indicated a significant interaction between probability and magnitude. F(6, 108) = 3.33, p < .05. This result supports the initial assumption made about the multiplicative combination process for probability and magnitude when a linear response function is assumed. No significant interaction was obtained between probability and time, F(9, 162) = .86, p > .05, or between time and magnitude, F(6, 108) = 2.01, p > .05. Therefore, when a linear response function is assumed, the dual distributive model described the patterns observed in the mean rating of desirability.

Following the procedure in Experiment 1, the ordinal consistency of the multiplicative, distributive, and dual distributive models (Equations 1a–1c) with the observed responses was compared under the assumption that the response function may be nonlinearly related to the implicit ratings (\( \psi_{pm} \)). Because of the similarity in response patterns across individual subjects and sessions, the mean ratings across sessions were averaged across subjects for the following analyses. The response function was parameterized using the integrated spline approach described in Experiment 1. Each model was tested using the same number of parameters and order for the spline function although knot placement varied according to the form of the function. For each model six parameters for the response function (including the intercept), three scale values for the magnitude of the return, four scale values for time, and two scale values for probability (the highest and lowest probability scale values were fixed at 1 and 1.0, respectively) were estimated.

A plot of the predicted and observed values for each model is shown in Figure 10 with the corresponding response function parameters and the vertical lines indicating the location of the knots. Each graph shows the predicted and observed values plotted as a function of the value of the investment derived from the model. The points correspond to the observed values for each investment. The lines indicate the predicted values. The percentage of variance accounted for by the dual distributive model (97.75%) and the distributive model (99.41%) was high but was exceeded by the percentage of variance accounted for by the multiplicative model (99.70%). Corresponding with the results obtained in Experiment 1, deviations from the predicted values of the dual distributive model were relatively large. The deviations from the distributive model are less pronounced than those observed in Experiment 1, though this could be attributed to the smaller number of data points. The multiplicative model appears to be most consistent with the observed means. The predicted values from this model are shown in Table 1 with the observed means for comparison. The form of these response functions replicates the forms obtained in Experiment 1.

The comparison F statistic described in Experiment 1 was also computed to evaluate the fit of the models. There were no significant deviations between the observed responses and the predictions generated by the multiplicative model, F(33, 48) = .46, p > .05. Unlike the results obtained in Experiment 1, no significant deviations were associated with the distributive model, F(33, 48) = .89, p > .05. But significant deviations were verified in the fit of the dual distributive model, F(33, 48) = 3.41, p < .05.

Preference model. The mean preference ratings obtained from each subject for two sessions were averaged across subjects. These means are shown in Figure 11 as points. The predicted values represented by the lines will be described later. The investments from Stimulus Set A were ordered according to the marginal means of the preference ratings and are given on the abscissa. Each line represents an investment from Stimulus Set B. The relation between the investments corresponds to the pattern of observed values obtained in Experiment 2. The stimuli that were included in the design for Experiment 2 and eliminated from the present design represented the most preferred and least preferred investments for Stimulus Set B. The elimination of these...
stimuli accounts for the less extreme bulge in the pattern of the means. One investment from Stimulus Set B that fell in the center of the pattern was also omitted. However, other modifications in interest values produced the same type of overlap in the current data.

As in Experiment 2, the barrel-shaped pattern in Figure 11 indicates that a difference model is inappropriate or that the response function relating the implicit comparisons ($p_{AB}$) to the observed responses ($P_M$) is nonlinear. In order to test the accuracy of the discounting preference model (Equation 4), an integrated second-order spline function (Winsberg & Ramsay, 1981) was used to parameterize the response function, $J_p$. Because the pattern of responses obtained from individual subjects was consistent with the means, the following analyses were performed on the group means.

Four segments were used to define the form of the response function using a second-order spline function. This formulation included 6 estimated parameters ($a_j, j = 1$ to 5 and an intercept, $J_0$) for the response function, 10 scale values for the stimuli from Set A (the lowest and highest scale values were fixed to define the scale unit), and 4 scale values for the stimuli in Set B. These values were estimated to minimize the sum of the 48 data-model squared discrepancies.

The results of the analysis indicated that the response function maximizing the fit of the discounting preference model is approximately S-shaped ($a_j = 1.55, .22, 2.01, .26, .28$). The form of this function is shown in Figure 12 (top panel), with the vertical lines indicating the location of the knot segments. The predicted values are represented by the solid line and the observed values are plotted as points. With this model 99.08% of the variance in preference ratings was represented. The predicted preference ratings are plotted in Figure 11 as lines, with the observed values represented by points. The predicted values correspond closely with the observed response pattern in the group means. There appears to be no systematic deviations in the predicted values.

A comparison $F$ statistic (as described in Experiment 1) indicated that the mean squared model-data discrepancy was about 2.37 times as large as the mean squared deviation between sessions. This ratio may be approximately $F$ distributed with (28, 48) degrees of freedom. This result indicates significant deviations ($p < .05$) in the fit of the difference model to the observed preference ratings. However, there appear to be no meaningful patterns in the deviations.

**Attribute comparison model.** The attribute comparison model was also tested to determine if the present set of data could be represented by this type of cognitive processing. This model was defined in Equation 7. As described in Experiment 2, this model assumes that the preference ratings are generated by adding the attribute comparison values. $J_p^*$ is assumed to be a monotonic transformation of the attribute composite and is parameterized
by the same number of knot segments as the discounting preference model to maximize the fit of the attribute comparison model.

The matrix of preference means was reorganized to include six levels of interest magnitude combinations across investments \((\theta_m)\) and eight time combinations \((\theta_t)\). In order to test the model defined in Equation 7, 18 parameters were estimated: four scale values for the magnitude of interest attribute (two scale values were fixed), eight scale values for the time attribute, six parameters for the nonlinear response function \(L_x\) including an intercept \(J_0\).

The predicted and observed data values are shown in Figure 12 (bottom panel). The attribute comparison model accounted for 97.08% of the variance in preference ratings. Comparing the pattern of deviations of the attribute comparison model with the discounting preference model shown in Figure 12 (top panel) clearly indicates that a wider range of deviations occurs using a model that assumes that comparisons were done across attributes. These results concur with the conclusion drawn in Experiment 2 that subjects compare the discounted values of the investments to produce these preference responses.

**Discounting function for magnitude and time.** The estimated scale values obtained from the discounting preference model (Equation 8) for Investment Sets A and B are shown in Figure 13. The amount of the interest to be gained (magnitude) is shown on the abscissa and each line represents a different time requirement. The scale values are the marginal means of the investment scale values \((\psi_m, \psi_t)\) after a linear transformation to be described later. The diverging bilinear fan pattern indicates that a ratio discounting operation describes the effect of time on the evaluation of both sets of investment returns. The form of this relation replicates the bilinear fan pattern obtained in Experiment 2.

**Scale value estimates.** The marginal means of the investment scale values representing each interest magnitude and time level obtained from the discounting preference model (i.e., from the preference task) were linearly transformed to match the minimum and maximum estimated scale values obtained from the multiplicative model of the risky investments. The psychophysical functions for the stimuli used in both tasks are shown in Figure 14. The lack of scale convergence between the scale values across tasks in Experiment 1 and 2 was replicated in this within-group task comparison. The scale values obtained for the magnitude of the return in the risky investment task exceeded the values estimated from the riskless investment preference task. The scale values for time that were estimated from the riskless investment preference task exceeded the scale values obtained from the risky investment rating task. These patterns replicate the patterns described for Experiment 1 and 2.

**Discussion**

The purpose of this experiment was to determine (a) whether the ratio discounting function would be supported using a different group of subjects and a within-subject manipulation of task, (b) whether the form of the nonlinear response functions would be consistent with the type of response scale used (e.g., bipolar and unipolar) regardless of the mode of presentation, and (c) whether scale convergence could be obtained when the same subjects completed both tasks.

The results obtained with the risky investment and preference task clearly replicated the results obtained in Experiments 1 and 2, respectively. When evaluating investment stimuli with positive outcomes, subjects discount the value of the return in proportion to the time required by the investment terms. Although the multiplicative model could not be discriminated as superior to the distributive model using a statistical criterion in this experiment, the reduction in the number of stimuli may account for the lack of power to do so.

The S-shaped response function associated with the preference task in Experiment 2 and the negatively accelerated response function obtained for the multiplicative model of the rating task were both replicated in this experiment. This replication eliminates the possibility that the response functions were determined by different groups of subjects. Furthermore, it is important to note that the method of presenting the response scales (paper and pencil vs. video screen) did not affect the form of these response functions. In Experiments 1 and 2, subjects used a line mark response on paper and the subjects in Experiment 3 responded by moving a cursor from the center position on a video screen.

Finally, the relation between the psychophysical scales obtained across the tasks in Experiment 1 and 2 was replicated when the
same group of subjects completed both tasks. These results indicate that the lack of scale convergence shown in these experiments cannot be attributed to group differences but appears to be related to the type of task used. Schoemaker (1982), in reviewing expected utility theory, describes the normative view that scale convergence cannot be obtained when the subjective values of stimuli derived from risky situations are compared with the subjective values obtained from riskless alternatives. The only empirical demonstration of this hypothesis cited was Tversky's (1967a) comparison of the value of cigarettes and candy in risky and riskless circumstances. His results indicated quite consistently over individual subjects that when these commodities were evaluated as part of a risky situation they had higher subjective values than the same commodities evaluated in riskless circumstances. The current results indicate that the interest to be gained in a risky investment was associated with higher subjective values than the interest gained from a riskless investment.

The psychophysical functions for time also varied across tasks. For the risky investments the correspondence between the physical and subjective values for time was nearly linear. For the riskless investments, the subjective values were a negatively accelerated function of the physical values for time. Because these patterns were replicated, it seems safe to conclude that people also evaluate the time factor differently if they are considering an explicitly risky versus a riskless future outcome.

Experiment 4

Mischel and Grusec (1967) proposed two possible interpretations for the effects of delay on the evaluation of outcomes. They noted that delayed outcomes could be perceived as implicitly risky. Such an account of the effects of delay implies that delayed positive outcomes are less preferred than immediate outcomes because delay is associated with the possibility that the outcome will not occur. Delayed negative outcomes would be preferred to immediate negative outcomes according to this interpretation, because they introduce the possibility that the negative event will not materialize. The explanation for the effects of delay as due to an implicit risk would be supported by a ratio discounting function. A ratio discounting function reduces the value of positive events and reduces the aversiveness of negative events. Mischel and Grusec (1967) also proposed that waiting for any type of outcome could be aversive and reduce the positive value or increase the aversive value of the event. This interpretation would be supported by a subtractive discounting function that reduces the value of outcomes as they are pushed into the future regardless of their positive or negative nature.

Several studies have been completed to determine how subjects respond to delayed aversive events. Mischel, Grusec, and Masters (1969) found that adults prefer immediate shock to delayed shock, which supports the aversive waiting (subtractive discounting model) hypothesis. Yates and Watts (1975) criticized the studies reported by Mischel et al. (1969), modified the procedure for establishing the total value of the outcome, and demonstrated that preference for immediate losses varies across individuals.

The results of the first three studies are compatible with both the implicit risk and the value reduction hypotheses proposed by Mischel and Grusec (1967). Given the multiplicative relation between explicit risk and magnitude, the ratio discounting function for time may represent an implicit risk for the subject. However, the subjective value of the outcome may also be viewed as reduced by time in proportion to its immediate worth, representing an aversive component to waiting. In order to discriminate these plausible interpretations, the type of outcome could be manipulated so as to change the ordinal characteristic of the utilities as a function of these interpretations (Yates & Watts, 1975). For example, if time delay is viewed as an implicit risk, the evaluation of future consequences that are negative in value would be rated as more desirable than aversive consequences that occur immediately. If, however, waiting per se is viewed as aversive, the type of consequence (i.e., positive or negative) would be rated in the same way as a function of time delay. Both events would be seen as reduced in desirability.

The purpose of Experiment 4 was to determine whether sub-
jects would use the same discounting process when evaluating stimuli that described negative consequences. In one task, subjects compared credit plans that varied in the amount of interest to be paid and in the length of time before the principle and interest had to be paid. An example stimulus is shown in Figure 15 (top panel). The credit plan provides a monetary event described as a function of time that parallels the riskless investments of the first three studies but frames the preference task for negative outcomes. In this case subjects will have to pay at the end of the designated time period. Both the discounting preference model (Equation 8) and the attribute comparison model (Equation 7) were tested using the data from the credit plan preference task.

In the second task, subjects evaluated the desirability of risky investments similar to the stimuli used in Experiments 1 and 3. In this case, however, the amount of interest to be gained was fixed and the possibility and amount of money that could be lost was varied. The amount of money that could be lost exceeded, equaled, or was less than the amount of interest that could be gained. The length of time required for the investment was also varied. An example stimulus is shown in Figure 15 (bottom panel).

Because the risky investments in this study are defined in relation to the amount of money that could be gained and lost, a model that incorporates both aspects of the investment will be tested. A model that can be used to test a ratio discounting function for these risky investments is given in Equation 13.

$$R_{\text{g/l}} = J[(1 - S_p)S_g/S_l] - [S_pS_g/S_l] + e_{\text{g/l}}. \quad (13a)$$

Ratings are described as a combination of the positive and negative attributes of the investments. All of the variables are first transformed to subjective values ($S_p$). $S_p$ represents the subjective probability of losing money, $S_g$ is the subjective value of the amount to be lost, $S_l$ is the subjective value of the amount to be gained, and $S_i$ is the subjectively evaluated length of time required to obtain a return. The first component of the model represents the positive aspect of the investment. $S_g$, the amount of money that could be gained is weighted by the subjective probability of the gain discounted by time. The second component of the model represents the negative aspect of the investment. $S_l$, the amount of money that could be lost is weighted by the subjective probability of the loss discounted by time. Both of these subcomponents use a multiplicative combination of probability and value discounted by the time interval required for the investment. These components are then assumed to add together to produce the overall value of the investment. This

![Figure 15](image-url)

Figure 15. (a) An example of the credit plan stimulus. (b) An example of the risky investment stimulus.
implicit value ($\phi_{wp}$) is assumed to be monotonically related to the observed ratings.

A critical test of the validity of the ratio discounting model for time and value is the presence of an interaction between positive and negative consequences. If time reduces the value of an outcome by a subtractive discounting operation, the effect of time would reduce the value or desirability of an outcome regardless of the type of consequence. If time reduces the value of an outcome by a ratio discounting process, time would reduce the value of a positive event and increase the value (i.e., reduce the negativity) of a negative event. By including both positive and negative components in these risky investments, the presence of this type of interaction can be assessed. This test provides a different type of evidence for the discounting function. If time reduces the value of a positive event and increases the value of a negative event, the discounting function must be ratio.

**Method**

**Preference task.** Subjects rated their degree of preference for one of two credit plans presented on each trial. An example of the video screen is shown in Figure 15 (top panel). The subject was instructed to act as a counselor who had a client who needed to borrow $1,000 in order to purchase some equipment. In order to simplify the task, subjects were told that their client would not be required to pay anything until the end of the time period, at which time the principal and all the interest were due. Subjects compared each set of credit plans and indicated their degree of preference for one of the credit plans by moving the cursor to the appropriate location on the response scale.

At the start of the session, subjects were asked whether they would prefer to pay back a loan in 3 months or 3 years. Subjects who are opposed to using credit preferred to terminate the loan as soon as possible. Based on these responses, the best and worst credit plan was defined as the anchor stimuli for these judgments. These anchor stimuli were shown at the bottom of the screen for each trial so that the subjects knew the range of credit plans that would be presented.

**Preference task design.** The 12 levels or credit plans for Stimulus Set A were constructed by a factorial combination of three interest payments ($60, $160, $320) and four time delays to the payment (3 months, 6 months, 1 year, 3 years). The four credit plans of Stimulus Set B were constructed from a factorial combination of two levels of interest to be paid ($100, $240) and two time delays (9 months, 2 years). Each of the 12 credit plans of Stimulus Set A was combined with each of the four credit plans for Stimulus Set B, yielding 48 ($12 \times 4$) preference trials.

**Risky investment task.** The rating task instructions are described in Experiment 4 with the following exceptions. The stimulus set included in this task varied in the amount of money that could be lost. Therefore each subject was first asked whether they would prefer to lose $320 in 3 months or in 3 years. Their response was used to define the temporal orientation of the anchor stimuli on the endpoints of the response scale. An example of the risky investments and the response scale is shown in Figure 15 (bottom panel).

**Risky investment design.** The 48 trials were constructed from a factorial combination of three levels of money that could be lost ($60, $160, $320), four time delays (3 months, 6 months, 1 year, 3 years) and four levels of the probability of losing money (.10, .30, .50, .80). The amount of money that could be gained was fixed at $160.

**Procedure.** The procedure was identical to the protocol described in Experiment 3.

**Subjects.** Twenty-five undergraduates from an introductory psychology course volunteered to participate for course credit. Ten randomly selected subjects completed the preference task first and 12 subjects completed the ratings of the risky investments first during each session.

![Figure 16. The predicted and observed preference responses plotted as a function of Credit Plan A. (Each line corresponds to a different credit plan from Set B. The lines indicate the predicted values derived from the discounting preference model. The observed means are shown as points.)](image-url)

**Results and Discussion**

**Preference task.** The mean preference ratings obtained from each subject for two sessions were averaged across subjects and are shown in Figure 16 as points. The predicted values represented by the lines will be described later. The credit plans from Stimulus Set A were ordered according to the marginal means of the preference ratings and are given on the abscissa. Each line represents a different credit plan from Stimulus Set B. The preference model defined in Equation 4 specifies that the relation among the credit plans should be described by parallel lines. The barrel-shaped pattern shown in Figure 16 is similar to the pattern obtained with the investment stimuli and indicates that either the difference preference model is inappropriate or the response function relating the implicit comparisons ($P_{AB}$) to the observed responses ($P_{AB}$) is nonlinear. An integrated second-order spline function was used to parameterize the response function ($f_P$) in order to test the adequacy of the difference preference model. Because the patterns of responses obtained from individual subjects were similar (see Footnote 3), the following analyses were performed on the means.

The model for the comparison of credit plans included 6 estimated parameters for the response function ($a_j, j = 1, 5$ and an intercept $J_0$), 10 scale values for Stimulus Set A (the lowest and highest scale values were fixed to define the scale unit), and 4 scale values for Stimulus Set B. The parameters were estimated to minimize the sum of the 48 data-model squared discrepancies.

---

3 One subject was dropped prior to the analyses because of a shift in response strategies across sessions. Two subjects were eliminated from the analyses because they indicated in their interviews that they never preferred to use credit.
The results of the analysis indicated that the response function maximizing the fit of the difference model is approximately S-shaped ($a_j = .34, .61, 2.81, .54, .27$). The form of this function is shown in Figure 17 (top panel) with the vertical lines indicating the placement of the knot segments. Predicted values are represented by the solid line and the observed values are plotted as points. The form of this function is consistent with the results obtained using investment stimuli. The discounting preference model accounted for 99.23% of the variance in the preference ratings. The predicted preference ratings are plotted in Figure 16 as lines with the observed values represented by points. The predicted values correspond closely with the observed response pattern in the group means. There appear to be no systematic deviations in the predicted values.

The comparison $F$ statistic described in Experiment 1 indicated no significant deviations in the fit of the difference preference model, $F(28, 48) = 1.14, p > .05$, using the variance across sessions as an estimate of error.

*Attribute comparison model.* A model assuming an attribute by attribute comparison process of the credit plans was also tested to determine if the present set of data could be better represented with this type of cognitive process. The general model was defined in Equation 7. According to this model, $\theta_a$ represents the comparison of the amounts of interests to be paid for the loans and $\theta_t$ represents the comparison of the time delay before the principal and interest are due. The preference ratings for each pair of credit plans are then determined by an additive combination of the attribute comparison values. $J^s$ is assumed to be a monotonic nonlinear transformation of the attribute composite that will be parameterized to maximize the fit of the attribute comparison model.

The matrix of preference means was reorganized to include six combinations of cost magnitudes across credit plans ($\theta_a$) and eight time combinations ($\theta_t$). For this analysis 18 parameters were estimated to test the attribute comparison model: four scale values for the cost magnitude attribute (two scale values were fixed), eight scale values for the time attribute, and six parameters for the nonlinear response function ($J^s$) including an intercept.

The predicted and observed data values are shown in Figure 17 (bottom panel). The attribute comparison model accounted for 99.11% of the variance in preference ratings. In comparison with the deviations shown in Figure 17 (top panel) for the discounting preference model, only a slightly wider range of discrepancies was obtained using a model that assumes comparisons were done across attributes. These deviations were not significant according to the appropriate $F$ statistic, $F(30, 48) = 1.24, p > .05$. According to these results, the discounting preference model provides a slightly better representation of the observed means visually and according to the total amount of the variance predicted. As with the investment stimuli, it appears that subjects compared the subjective values of the credit plans rather than the individual attributes.

*Discounting function for magnitude and time.* The estimated scale values obtained with the discounting preference model (Equation 8) for Stimulus Set A and B are plotted in Figure 18 as a function of the cost of the loan with a different line for each time delay to the payment. The scale values for each interest charge and time delay are the marginal means of the credit plan scale values after a linear transformation that will be described later. The converging bilinear fan pattern indicates that a ratio discounting operation describes the effect of time on the value of the credit plans. Unlike the positive outcomes represented by the investment stimuli, credit plan scale values decrease as the cost of the loan increases and increase as the time delay to the payment gets longer. The value of the credit plan is represented by the length of time allowed per dollar payment required. Therefore, paying $100 for a 1-year loan is viewed as equivalent to spending $300 for a 3-year loan if the psychophysical functions for time and cost magnitude are both identity functions. The same converging pattern was obtained for both sets of credit plans.

*Risky investment task.* The mean ratings obtained for each investment, averaged across sessions and subjects, are shown in
Figure 18. The scale values derived from the discounting preference model for Credit Plan Sets A and B plotted as a function of the scale values (S) of the cost of the loan. (Each line corresponds to a different time delay to payment.)

Table 2. For each delay there is a divergence in the ratings as the amount to be lost increases with the probabilities .1, .3 and .5, whereas the ratings converge across increases in the magnitude of loss when the probability of losing is higher (.5 and .8). Unlike the ANOVA results obtained with positive consequences in risky investments, the results of the ANOVA of these investments indicated significant interactions between time and amount to be lost, $F(6, 126) = 2.91, p < .05$, time and probability of loss, $F(9, 189) = 6.16, p < .05$, and the probability of loss and loss magnitude, $F(6, 126) = 14.32, p < .05$. The three-way interaction was also reliable, $F(18, 378) = 1.77, p < .05$, supporting a multiplicative model for time, loss magnitude, and probability of loss.

As in the previous studies, the initial assumption that probability and loss magnitude combine multiplicatively is supported when a linear response function is assumed. Unlike the previous results, which supported a multiplicative model only when the response function was assumed to be negatively accelerated, support for a ratio temporal discounting model was obtained under the assumption that the response function was linear. In order to determine if the response function differed for these risky investments and to maximize the fit of the temporal discounting model described in Equation 13, the following analysis was completed.

The purpose of this analysis was to assess the accuracy of the temporal discounting model described in Equation 13 in describing the pattern of observed responses. In this model, the response function may be a nonlinear function relating the implicit ratings ($\psi_{ik}$) to the observed ratings ($R_{ik}$). Similar response patterns were obtained for the individual subjects (see Footnote 3) and across sessions so the mean ratings were used for this analysis. The response function was parameterized using the integrated spline function described in Experiment 1. Sixteen estimated parameters were used to fit the discounting model in Equation 13: six estimated parameters for the response function (including an intercept), three scale values for the amount of money that could be lost, four scale values for the time delay, two scale values for the probability of losing money (the lowest and highest probability scale values were fixed at .1 and .8, respectively), and one scale value for the amount of money that could be gained.

A plot of the predicted and observed values for the multiplicative discounting model is shown in Figure 19 (top panel) with

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Table 2. The Observed Means (O) and the Predicted Values (P) From the Multiplicative Model
TEMPORAL DISCOUNTING OF OUTCOMES

the vertical lines representing the location of the knots. The predicted values represented by the line and the observed values, plotted as points, are shown as a function of the value of the investment described by the model. The multiplicative model accounted for 99.56% of the variance in the ratings. The comparison $F$ statistic described in Experiment 1 indicated no significant deviation between the observed values and the predicted values derived from the multiplicative discounting model, $F(32, 48) = .73, p > .05$.

These risky investments differed from the type of investment described in Experiments 1 and 3. In the former studies, the investments were described only in relation to the time required and the probability of gaining various returns. The response function for these investment ratings was clearly negatively accelerated. In contrast, the current study varied the time factor and the probability of losing different amounts of money. The gain was fixed in terms of the dollar amount but varied in value when viewed in relation to the time required to obtain it (e.g., gain/time). The form of the response function for these investments was positively accelerated.

The scale used to rate these investments ranged from undesirable to desirable; however, the subjects were evaluating the desirability of investments that varied in the amount of money that could be lost. The form for this response function is the inverse of the response function obtained with investments that varied in the amount to be gained. If the subjects had changed their reference point from the left to the right anchor for this task, the inverse form of the response function might be reasonable. The inverse response scale would represent a "scale" of the degree of aversion. In order to test this hypothesis the following analysis was computed.

The observed response scale was reversed and the terms in Equation 13a were adjusted to account for the change in response orientation:

$$60 - R_{\text{gain}} = J[60 + (S_p S_g / S_t)] - [(1 - S_p) S_g / S_t] + e_{\text{gain}}. \tag{13b}$$

This adjusted model was fit using the transformed responses and the same number of parameters described in the previous analysis. The form of the resulting response function is shown in Figure 19 (bottom panel). By reversing the scale to correspond to the degree of aversion, the negatively accelerated function obtained in the previous experiments was replicated.

Although the current design varied the amount of money that could be lost, some investments were more positive or likely to lead to a gain than others. In order to determine whether time would have a different effect on investments that were more likely to produce a gain than on investments that were more likely to lead to a loss, changes in the mean ratings across time were compared when the probability of losing $320 was .8 and the probability of losing $60 was .1. The first condition represents the investments most likely to lead to a large loss and the second condition those investments least likely to lead to a relatively small loss. The means for these investments across time are shown in Table 3. In general, the desirability of the investment increases into the future if a loss is expected, whereas the desirability of the investment decreases into the future if a gain is more likely than a loss. This type of interaction supports a ratio discounting function for time and clearly contradicts the expected pattern of results that a subtractive discounting function would produce.

Table 3

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Figure 19. The predicted and observed rating responses plotted as a function of the discounting model. (Top panel: The response function obtained with the scale ranging from the worst to the best investment. The knots for the spline function were placed at $[-40, -40, -20, 0, 11, 30, 30]$. The response function parameter estimates were $[1.19, 0.56, 1.50, 1.66]$. Bottom panel: The response function obtained with the scale ranging from the best to the worst investment. The knots for the spline function were placed at $[-50, -50, -20, -10, 50, 200, 200]$. The response function parameter estimates were $[.15, .71, .70, .13, 0]$.}


Figure 20. The psychophysical functions for the magnitude of the loss and time derived from riskless and risky stimuli.

Scale value estimates. The marginal means of the credit plan scale values representing each magnitude and time level obtained from the discounting preference model were linearly transformed to match the minimum and maximum estimated scale values obtained from the ratio discounting model for risky investments. The psychophysical functions for the stimuli used in the two tasks are shown in Figure 20. The scale values obtained for the cost of the loan in the preference task exceeded the scale value estimates obtained with the risky investment stimuli. This is the inverse of the relation found between the scale values for the magnitude of the return in the positive outcome studies. With positive consequences, the scale values derived from the risky stimuli exceeded the values obtained with the riskless investment stimuli. The psychophysical functions obtained for the time scale also differed across tasks. The scale values obtained with the risky investment stimuli exceeded the scale values obtained with the credit plans. With positive consequences, the riskless stimuli were associated with higher time scale values than were the risky investments.

General Discussion

The purpose of this series of studies was to establish converging empirical support for a temporal discounting model using two different types of judgment tasks. Consistent support for a ratio discounting function for time was obtained using riskless and risky investment stimuli (and credit plans) featuring positive or negative consequences. When subjects were asked to rate their preferences for riskless investments or rate the desirability of risky investment stimuli (and credit plans) featuring positive or negative consequences. When subjects were asked to rate their preferences for riskless investments or rate the desirability of risky investments that varied in the magnitude of the return and the time required, subjects preferred immediate gains, and the amount to be gained was discounted in proportion to the delay. When subjects were asked to rate their preferences for credit plans or rate the desirability of risky investments that varied in cost and in the delay to payment, the inverse relation was obtained. In response to the credit plans, subjects preferred to delay payments and discounted the amount to be paid in proportion to the delay. When risky investments were used to vary the amount of money that could be lost, an interaction was obtained. If the probability of losing the money was high, so that the expected value was negative, subjects rated the investments as more desirable the longer the time to maturity. If the probability of losing was low, so that the expected value was positive, subjects rated the investments as less desirable the longer the time to maturity. In both cases, the positive and the negative outcomes were discounted in proportion to the delay. Thus, the following ratio discounting function was consistently supported in this series of studies:

\[ \psi_{pmt} = S_p S_{m} / S_{t}, \]  

where \( S_p \) represents the subjective probability of the event, \( S_m \) represents the subjective magnitude of the outcome, and \( S_t \) represents the subjective time scale. The ratio discounting function indicates that the effect of delay on these monetary decisions may be interpreted as an implicit risk factor by the subjects. These results are incompatible with the hypothesis that time adds an aversive waiting component to the investment outcomes.

In the course of comparing and evaluating the response patterns obtained from different designs and tasks, two aspects of a general model were also verified across experiments. An approximately S-shaped nonlinear transformation maximized the fit of the discounting preference model to the observed values. The same type of transformation was required whenever a bipolar response scale was used. The form of this transformation implies that subjects calibrate their responses to the modeled values more closely near the center of the bipolar scale (i.e., near the indifference reference point) than near the ends of the scale. Near the ends of the response scale (at either extreme), the observed ratings are more similar than the values predicted by the discounting preference model. With the unipolar rating scale, a negatively accelerated transformation maximized the fit of the multiplicative model to the observed values. This type of function represents the same type of response pattern described for the bipolar scale except that the reference point lies at one end of
the scale. With the unipolar scale, those investments viewed as most desirable (or undesirable) are viewed as more similar by the subjects than would be predicted by the ratio discounting model. Stimuli rated near the reference point of the scale correspond more closely to the predicted values generated from the discounting function. The response function in the general model is critical in providing a means of comparing the responses of subjects across tasks. If the response functions had not been modeled, both the ratio and the subtractive models would have been supported for the risky investment task depending on the type of outcome that was manipulated. These results may have some implications for the different models supported in the studies reported by Örlandahl and Sjöberg (1979) when different response measures were compared.

The last issue is scale convergence across tasks. With positive outcomes, the scale values obtained for the amount of money to be gained in the risky investments exceeded the scale values obtained with the riskless investments. This result was first obtained across subject samples but was replicated using a within-subject design. This result corresponds with the empirical results obtained by Tversky (1967a) and supports the economic theories (Schoemaker, 1982) that assume that utility scales of magnitudes constructed under certainty are only monotonically related to the utility scales of magnitudes constructed under risk. When negative outcomes are evaluated, the inverse relation is obtained.

In that case, the subjective values for the amount of money to be paid are higher when derived from a riskless situation than from risky investments. The current results extend the empirical evidence for the lack of linear scale convergence to differences in subjective values of time obtained from risky and riskless stimuli. The inverse relation between scale values is obtained for time in riskless and risky stimulus sets. The subjective values for time when evaluating positive consequences from riskless investment stimuli exceed the subjective values obtained for time when evaluating risky investments. With negative consequences, temporal scale values for risky investments are higher than temporal scale values for riskless credit plans.

Situational factors may play an important role in determining how time depreciates value. In some situations, the notion of an implicit risk factor is evident. For example, in planning a career unforeseen circumstances may block the possibility of completing the training program and obtaining the job. Or for an investment decision, circumstances could develop that would produce an immediate need for the money that was invested. In other situations, however, the future outcome may be close to certain regardless of intervening events. For example, long-range environmental costs have often been ignored when the impact of the course of action selected was clearly predictable. It is unlikely in such cases that the time delay is viewed as an implicit risk factor. Therefore, the possibility of different discounting processes must be evaluated in the context of the relevancy of implicit risk.

References
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