HW #1 Solutions

Saturday, January 29, 2005
12:50 AM

1.1 a. nominal
b. ordinal
c. interval
d. nominal

1.2 a. $Y \sim bin(100, \frac{1}{4})$

b. $E(Y) = np = 25$, $Var(Y) = np(1-p) = 18.75$

c. $\begin{pmatrix} n_1 \\ n_2 \\ n_3 \\ n_4 \end{pmatrix} \sim \text{multinomial}\left(100, \begin{pmatrix} .25 \\ .25 \\ .25 \\ .25 \end{pmatrix}\right)$

d. $E(n_j) = 25$, $var(n_j) = 18.75$

$cov(n_j, n_k) = -np_j p_k = -100 \cdot \frac{1}{4} \cdot \frac{1}{4} = -6.25$

$corr(n_j, n_k) = \frac{-6.25}{18.75} = .333$

1.3 mixture

1.4 $X_i \sim iid \text{ Bernoulli}(1/6) = Bin(1, 1/6)$

a. $P(X_1 = 0, \dots, X_6 = 0)$

$= P(X_1 = 0)^6 = \left(\frac{5}{6}\right)^6 = .335$

b. $P(Y = y) = P(X_1 = 0, \dots, X_{y-1} = 0, X_y = 1)$

$= \prod_{j=1}^{y-1} P(X_j = 0)\, P(X_y = 1)$

$= \left(\frac{5}{6}\right)^{y-1} \frac{1}{6}$ (called Geometric$(\frac{1}{6})$ distribution)

1.5 $X \sim bin(n, \pi)$

$n = 842 + 982 = 1{,}824$
$x = 842$

$\Rightarrow \hat{\pi} = \frac{x}{n} = \frac{842}{1824} = .462$

$H_0: \pi = .5$

Wald: $z_W = \frac{\hat{\pi} - \pi_0}{\sqrt{\hat{\pi}(1-\hat{\pi})/n}} = \frac{.462 - .5}{\sqrt{.462(1-.462)/1824}}$

$= -3.26$ P-value $= P(|Z| > 3.26) \approx 0$

C.I. $\hat{\pi} \pm z_{\alpha/2}\sqrt{\frac{\hat{\pi}(1-\hat{\pi})}{n}}$

Score: $z_S = \frac{\hat{\pi} - \pi_0}{\sqrt{\pi_0(1-\pi_0)/n}} = \frac{.462 - .5}{\sqrt{.5(1-.5)/1824}} = -3.25$

C.I. [illegible]

$\sqrt{\pi_0(1-\pi_0)/n}$ $\sqrt{.5(1-.5)/1824}$

C.I. Use eqn on p.16