5.1 Parameter Interpretation

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3:48 PM

$$\pi(x) = P(Y=1 \mid X=x) = \frac{\exp(\alpha+\beta x)}{1+\exp(\alpha+\beta x)}$$

The log odds (logit) is linear

$$\text{logit}[\pi(x)] = \log\frac{\pi(x)}{1-\pi(x)} = \alpha+\beta x$$

5.1.1. Interpreting β

- the odds increase multiplicably by e^{β} for every 1-unit increase in x
- other interpretation?

$$\frac{\partial}{\partial x}\pi(x) = \beta\frac{\pi(x)}{1-\pi(x)}$$

The steepest slope occurs at x for which $\pi(x)=1/2$ or at $x = -\frac{\alpha}{\beta}$ (why?)

called the "median effective level"

EL 50

In toxicological studies, called LD 50.

lethal dose

- Or, report $\pi(x)$ values at quartiles of x

5.1.2. Looking at the data

- plot(x, y) doesn't help much.

- If x has only a few "settings", draw the "sample logit"

$$\log\left[\frac{p_i}{1-p_i}\right] \quad \text{where } p_i = \frac{y_i}{n_i}$$

the "sample logit"

$$\log\left[\frac{p_i}{1-p_i}\right] = \log\left[\frac{y_i}{n_i - y_i}\right]$$

where n_i, y_i are # of obs. and # of '1's at setting i of x. and $p_i = \frac{y_i}{n_i}$

Or, the adjusted values

$$\log \frac{y_i + \frac{1}{2}}{n_i - y_i + \frac{1}{2}}$$

$y_i = 0 \Rightarrow \log \frac{\frac{1}{2}}{n_i + \frac{1}{2}}$

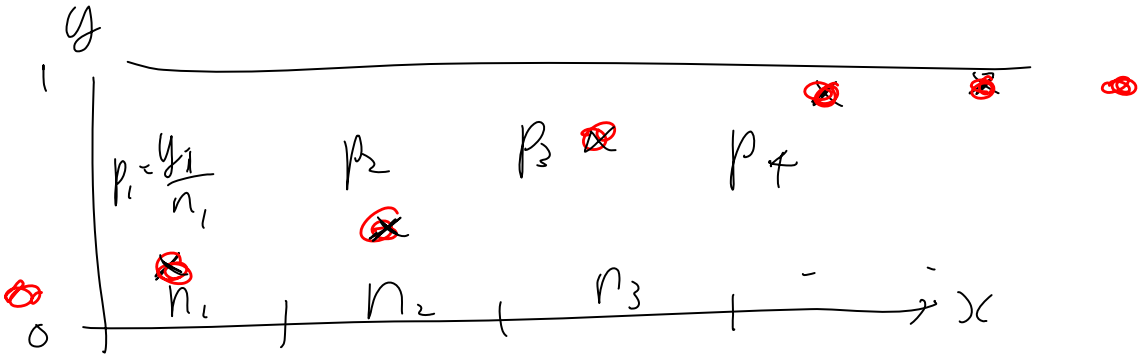
- plot(x, sample logit) should be _____.
- If X is cts and all $n_i = 1$, what to do?
  - Grouping the data
  - Smoothing
    e.g. generalized additive model (§4.8)
    should observe S shaped curve

5.1.3. Crab example

$$\hat{\pi}(x) = \frac{\exp(-12.351 + 0.497x)}{1 + \exp(-12.351 + 0.497x)}$$

$$EL_{50} = -\frac{\alpha}{\beta} = -\frac{-12.351}{.497}$$

$$=$$


y
1
$p_1 = \frac{y_1}{n_1}$
p_2
p_3
p_4
n_1
n_2
n_3
0
x