1 Basic probability

1.1 Sample spaces and events

- A "sample space," is the set of all possible outcomes of an experiment. The sample points are the elements in a sample space.
- An "event" in a subset of the sample space
- Two events are mutually exclusive if …

1.2 Assignment of probabilities

- Probability axioms
- $P(A \cup B)$=___
- $P(A^c)$=___

1.3 Simulation of events on the computer

- How do you simulate a coin tossing on a computer?

1.4 Counting techniques

- Multiplication rule
- Permutation of n distinct items taken r at a time
- Combination: an unordered arrangement of r items selected from n distinct items

1.5 Conditional probability

- $P(B|A)$=___
- Multiplication rule $P(A \cap B)$=___
- Law of total probability: if $E_1,\ldots,E_k$ are a partition of the sample space, then P(A)=___

1.6 Independent event

- Events A and B are independent if ___
- Events $E_1,E_2,\ldots$ are mutually independent if ___

2 Discrete random variables

2.1 Random variables

- The probability mass function p(x) satisfies ___

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- CDF $F(x)=P(X \le x)$

2.2 Joint distributions and independent random variables

- The joint probability mass function p(x,y)=P(X=x,Y=y)
- The marginal probability mass function $p_X(x)$=___
- The conditional probability mass function P(X=x|Y=y)
- Discrete random variables X and Y are independent if ___
- Discrete random variables $X_1,X_2,\ldots,X_n$ are mutually independent if ___

2.3 Expected values

- E(X)=___
- If Y is a function of a random variable X, then E(Y)=E(Q(X))=___
- E(a+bX)=___
- If $Y=g_1(X)+\ldots+g_2(X)$, then E(Y)=____

2.4 Variance and standard deviation

- VAR(X)=___ = ____
- STD(X)=___
- Chebychev's inequality
- VAR(a+bX) = ___
- STD(a+bX)=___

2.5 Sampling and simulation

- Random variables $X_1,\ldots,X_n$ are a random sample if they are i.i.d.
- Empirical probability distribution; empirical probability mass function

2.6 Sample statistics

- Sample mean $\bar{X}$=___
- Sample variance $S^2=\frac{1}{n}\sum_{i=1}^{n}(X_i-\bar{X})^2$
- Sample standard deviation S=___

2.7 Expected values of jointly distributed random variables and the law of large numbers

- E(Q(X,Y))=___

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- E(aX+bY)=___
- $E(a_1X_1+a_2X_2+\ldots+a_nX_n)$=___
- If X and Y are independent, E(XY)=___
- If X and Y are independent, VAR(aX+bY)=___
- If $X_1,\ldots,X_n$ are independent, $VAR(a_1X_1+\ldots+a_nX_n)$=___
- If $X_1,\ldots,X_n$ iid with $E(Xi)=\mu$ and $VAR(Xi)=\sigma^2$, then $E(\bar{X}), VAR(\bar{X}), STD(\bar{X})$ = ___
- Weak law of large number: under the i.i.d. setup as above, $\lim_{n\to\infty} P(|\bar{X}-\mu|>\varepsilon)=0$ for any $\varepsilon>0$.

2.8 Covariance and correlation

- COV(X,Y)=___
- CORR(X,Y)=___
- VAR(aX+bY)=___
- Bivariate random sample
- The sample covariance cov(X,Y)
- The sample correlation corr(X,Y)

2.9 Conditional expected values

- E(Y|X=x)=___
- E(Y)=E(E(X|Y))
- VAR(Y|X=x)=___
- VAR(Y)=E[___]+VAR[___]

3 Special discrete random variables

3.1 Binomial random variable

- A binomial random variable is ___
- For Y~bin(n,p), p(y)=___. Also, E(Y)=___ and VAR(Y)=___

3.2 Geometric and negative binomial random variables

- The geometric random variable is ____. We write it as X~Geo(p). In that case, p(x)=___, E(X)=___, VAR(X)=___
- Given an integer k>1, the negative binomial random variable is ___. We write it as Y~Nbinom(k,p). In that case, p(x)=___, E(X)=___, VAR(X)=___

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3.3 Hypergeometric random variables

- Consider a jar containing the total of N balls, r of which red, the remaining white. Randomly selecting n balls from the jar WOR, the number of red balls in the sample X~Hypergeo(N,r,n). In that case, p(x)=___, E(X)=___, VAR(X)=___
- Approximately binomial with p=___

3.4 Multinomial random variables

- Let $Y_1,\ldots,Y_k$ denote the number of times the mutually exclusive outcomes $C_1,\ldots,C_k$ occur in n iid trials. Let $p_i=P(C_i)$ for i=1,…,k. Then we say $(Y_1,Y_2,\ldots,Y_k)$~multinom(n, $p_1,\ldots,p_k$) and $P(Y_1=y_1,\ldots,Y_k=y_k)$=______ where ____
- $E(Y_i)$=__ and $VAR(Y_i)$=___
- $COV(Y_s,Y_t)$=____ if $s \ne t$.

3.5 Poisson random variables

- X~Poisson(μ) if p(x)=___
- E(X)=___ and VAR(X)=___
- If X~bin(n,p), P(X=x) ~ p(x) of Poisson(μ) if ____ and μ=__

3.6 Moments and moment-generating functions (MGF)

4 Markov chains

4.1 Introduction: modeling a simple queuing system

- A queue is a waiting line
- Consider the following queuing system:
  - A company has an assigned technician to handle service for 5 computers.
  - Each computer independently fails with probability.2 during any day
  - The technician can fix one computer per day.
  - If a machine breaks down, it will be fixed the day it fails provided there is no backlog. If there is a backlog, it will join a service queue and wait until the technician fixes those ahead of it.