Stat 6310 Spring 2006 Jaimie Kwon

Quiz #2 (50 minutes) **Name**_____

- This quiz is worth 21 points with the point values for the parts indicated. Be sure to show your work for full credit. Write your numerical answers in fractions or round them to the 3rd significant digits.

Bill receives average of 48 emails **a day**. Suppose we model the number of emails he receives as a time-homogeneous Bernoulli counting process with the frame size of 1 minute.

a) Find the rate λ in units of emails per hour. Also find the probability P of an email arriving in a single, one-minute frame.

$\lambda =$

P =

b) Let X be the number of emails Bill receives over **one hour**. Compute the mean and standard deviation of X (hint: what's the distribution of X?) (3 pt)

E(X) =

SD(X) =

c) Let Y be the number of 1-minute frames from one email to the next. Compute the mean and standard deviation of Y (hint: what's the distribution of Y?) (3 pt)

E(Y) =

SD(Y) =

- For questions d) ~ f) below, we model the number of emails Bill receives as a Poisson process by considering infinitely small frames. We still assume that he receives average of 48 emails a day.

d) Let N be the number of emails Bill receives over one hour. Compute the mean and standard deviation of N (hint: What's the distribution of N?) (3 pt)

E(N) =

SD(N) =

e) Let T be the time in hours between two successive emails. Find the P(T > 1 hour). (3 pt)

f) Let W_2 be the time in hours from some time 0 to the second email Bill receives. Find the P(W_2 > 1 hour). (3 pt)

g) Find the mean and standard deviation of W_2. (3 pt)

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Solutions to Quiz #2

a) λ = **2** emails/hour and p = $\lambda \Delta$ = 2*1/60 = **1/30**

b) X ~ bin(60, 1/30); E(X) = **2**, V(X) = 60*1/30*(1-1/30) = 1.9 and SD(X) = sqrt(1.9) = **1.39**

c) Y ~ Geo(1/30); E(X) = 1/p = **30** ; V(X) = (1-1/30)/(1/30)^2 = 870; SD(X) = sqrt(30) = **29.5**

d) N ~ Poisson(2); E(X) = **2**; V(X) = 2; SD(X) = sqrt(2) = **1.414**

e) T ~ Exp(2); P(T > 1 hour) = 1- F(1) = exp(-2) = **0.135**

f) $P(W_2 > 1) = P(N(1) \leq 1) = P(N(1) = 0) + P(N(1) = 1)$ = exp(-2) + exp(-2)*2 = 0.135 + 0.271 = **0.406**

g) W_2 ~ Gamma(2, 1/2); $E(W_2)$ = 2*1/2 = **1** (hour) ; $V(W_2)$ = 2*1/2^2 = 1/2; $SD(W_2)$ = sqrt(1/2) = **0.707**