Stat 6310 Spring 2006 Jaimie Kwon

Quiz #1 (50 minutes) **Name**____________________

- This quiz is worth 30 points with the point values for the parts indicated. Be sure to show your work for full credit. Write your numerical answers in fractions or round them to the 3^{rd} significant digits.

Suppose that the April weather in the Bay Area can be modeled as a two-state time-homogeneous Markov chain $\{X(n): n \geq 0\}$ with state space S = {0,1}, where X(*n*) =0 if it's sunny and X(*n*)=1 if it's rainy on day *n*. The initial distribution is $\underline{\pi}(0) = (0.3, 0.7)$. The one-step transition matrix is given by

$$P = \begin{matrix} 0 \\ 1 \end{matrix} \begin{bmatrix} .5 & .5 \\ .2 & .8 \end{bmatrix}$$

a) Suppose that on day 0, it was sunny (like today). Find the conditional probability that in the next three days, the weather will be rainy, rainy, and then sunny. That is, find P(X(1)=1, X(2)=1, X(3)=0|X(0) =0). (+5)

b) Find the unconditional "path" probability P(X(0)=0, X(1)=1, X(2)=1, X(3)=0). (+5)

c) Find $\underline{\pi}(1)$, the probability distribution of X(1). (+5)

d) The third power of *P* is $P^3 = \begin{matrix} 0 \\ 1 \end{matrix} \begin{bmatrix} .305 & .695 \\ .278 & .722 \end{bmatrix}$. Find the probability that it rains on the third from day 0, i.e. find P(X(3)=1). (Remember that the initial state is random with the initial probability distribution $\pi(0)$ given above.) (+5)

e) Find the steady state probability vector $\underline{\pi}$. (+5)

f) If it rains on a particular day, what's the expected number of days until it rains again? (+5)

Solutions to Quiz #1

$\underline{\pi}(0) = (0.3, 0.7)$ and $P = \begin{matrix} 0 \\ 1 \end{matrix}\begin{bmatrix} .5 & .5 \\ .2 & .8 \end{bmatrix}$

a) P(X(1)=1, X(2)=1, X(3)=0|X(0) =0)
   = P(0→1) P(1→1) P(1→0) = .5*.8*.2
   = **.08**

b) P(X(0)=0, X(1)=1, X(2)=1, X(3)=0)
   = P(X(1)=1, X(2)=1, X(3)=0|X(0) =0) P(X(0) =0) = .08*.3= **.024**

c) $\underline{\pi}(1)=\underline{\pi}(0)P$
   = [.3*.5+.7*.2 , .3*.5+.7*.8]
   = **(.29, .71)**

d) $\underline{\pi}(3)=\underline{\pi}(0)P^3$
   = [.3*.305 + .7*.278 , .3*.695 + .7*.722]
   = (.286, .714).
   The probability that it rains three days from day 0 is **.714**

e) We need to solve $\underline{\pi}P=\underline{\pi}$ (part 1) and $\pi_1+\pi_2=1$ (part 2).
   From the first part, we have
   $.5*\pi_1 + .2*\pi_2 = \pi_1$,
   or
   $.2*\pi_2 = .5 * \pi_1$. (The second column leads to an identical formula)
   Plugging $\pi_2 = 1- \pi_1$ (part 2) into the above, we have
   $.2 (1-\pi_1)= .5*\pi_1$,
   or
   $.2 = .7*\pi_1$.
   Thus, . π_1 = .2/.7 = .286 and π_1=5/7 = .714. The answer is **π=(.286, .714**)

f) $E(T_1|X(0)=1) = 1/\pi_1 = 7/5 =$ **1.4** (days) (+5)