Stat 6310 Spring 2006 Jaimie Kwon

## Midterm Solution

**1.**

a) S = {0, 1, 2, 3}; $P = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 1/3 & 0 & 2/3 & 0 \\ 0 & 2/3 & 0 & 1/3 \\ 0 & 0 & 1 & 0 \end{bmatrix}$

b) S = {0, 1, 2, 3}; dbinom(0:3, 3, .2)
$P(0 \to 0)$ = P(no compuer out of 3 breaks down) = dbinom(0, 3, .2) = .512
$P(0 \to 1)$ = P(one compuer out of 3 breaks down) = dbinom(1, 3, .2) = .384
$P(0 \to 2)$ = P(two compuers out of 3 break down) = dbinom(2, 3, .2) = .096
$P(0 \to 3)$ = P(three compuers out of 3 break down) = dbinom(3, 3, .2) = .008

$P(1 \to 0)$ = P(no compuer out of 2 breaks down) = dbinom(0, 2, .2) = .64
$P(1 \to 1)$ = P(one compuer out of 2 breaks down) = dbinom(1, 2, .2) = .32
$P(1 \to 2)$ = P(two compuer out of 2 breaks down) = dbinom(2, 2, .2) = .04
$P(1 \to 3) = 0$

$P(2 \to 0) = 0$
$P(2 \to 1)$ = P(no compuer out of 1 breaks down) = dbinom(0, 1, .2) = .8
$P(2 \to 2)$ = P(one compuer out of 1 breaks down) = dbinom(1, 1, .2) = .2
$P(2 \to 3) = 0$

$P(3 \to 0) = 0$
$P(3 \to 1) = 0$
$P(3 \to 2) = 1$
$P(3 \to 3) = 0$

So, $P = \begin{bmatrix} .512 & .384 & .096 & .008 \\ .64 & .32 & .04 & 0 \\ 0 & .8 & .2 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$

**2.**

a) $P(X(1) = 1, X(2) = 2 | X(0) = 0) = P(0 \to 1)\, P(1 \to 2) = 1/2 * 3/4 =$ **3/8**

b) $P(X(0) = 1, X(1) = 2, X(2) = 0) = P(X(1) = 2, X(2) = 0 | X(0) = 1)\, P(X(0) = 1)$
$= P(1 \to 2)\, P(2 \to 0)\, P(X(0) = 1) = 3/4 * 1 * .3 =$ **.225**

c) $\underline{\pi}(0)\, P = [.4, .3, .3] \begin{bmatrix} 0 & 1/2 & 1/2 \\ 1/4 & 0 & 3/4 \\ 1 & 0 & 0 \end{bmatrix}$ = c( .3*1/4 + .3*1,  .2,  .4*1/2+.3*3/4 ) = **(.375, .200, .425)**

d) Rewriting $(\pi_0, \pi_1, \pi_2)\begin{bmatrix} 0 & 1/2 & 1/2 \\ 1/4 & 0 & 3/4 \\ 1 & 0 & 0 \end{bmatrix} = (\pi_0, \pi_1, \pi_2)$, you have:

(*) $1/4 \times \pi_1 + \pi_2 = \pi_0$
(**) $1/2 \times \pi_0 = \pi_1$
$1/2 \times \pi_0 + 3/4 \times \pi_1 = \pi_2$
as well as
(***) $\pi_0 + \pi_1 + \pi_2 = 1$ .
Combining (*) and (**) gives
$7/4 \times \pi_1 = \pi_2$
and putting this and (**) into (***), we have
$2 \times \pi_1 + \pi_1 + 7/4 \times \pi_1 = 1$
or $(19/4) \times \pi_1 = 1$, i.e.
$\pi_1 = 4/19$
and
$\pi_0 = 8/19$ and $\pi_2 = 7/19$
follow. To summarize, $(\pi_0, \pi_1, \pi_2)$= **(8/19, 4/19, 7/19)**

e) $\pi_1$ = **4/19**, or about 21% of days

f) $1/\pi_0$ = **19/8 (days)**

**3.**

| | [,1] | [,2] | [,3] | [,4] | [,5] |
|---|---|---|---|---|---|
| [1,] | 1.0 | 0.0 | 0.0 | 0.0 | 0.0 |
| [2,] | 0.5 | 0.0 | 0.5 | 0.0 | 0.0 |
| [3,] | 0.0 | 0.5 | 0.0 | 0.5 | 0.0 |
| [4,] | 0.0 | 0.0 | 0.5 | 0.0 | 0.5 |
| [5,] | 0.0 | 0.0 | 0.0 | 0.0 | 1.0 |

a) P is given above:

b) $P^{(5)}(10 \to 5)$ = **0.125**

c) $P^{(5)}(10 \to 5) + P^{(5)}(10 \to 10) + P^{(5)}(10 \to 15)$ = 0.125 + 0 + 0.125 = **0.25**

d) Then circle the submatrices R and Q and identify them with letters. (+5)

| | [,1] | [,2] | [,3] | [,4] | [,5] |
|---|---|---|---|---|---|
| [1,] | 1.0 | 0.0 | 0.0 | 0.0 | 0.0 |
| [2,] | 0.0 | 1.0 | 0.0 | 0.0 | 0.0 |
| [3,] | 0.5 | 0.0 | 0.0 | 0.5 | 0.0 |
| [4,] | 0.0 | 0.0 | 0.5 | 0.0 | 0.5 |
| [5,] | 0.0 | 0.5 | 0.0 | 0.5 | 0.0 |

e) $\underline{\mu} = (I - Q)^{-1}\,\underline{1}$ = [3, 4, 3]; $\mu_{\$10} = 4$

f) $F = (I - Q)^{-1} R = \begin{bmatrix} 1.5\times 0.5 & .5\times 0.5 \\ 1\times 0.5 & 1\times 0.5 \\ 0.5\times 0.5 & 1.5\times 0.5 \end{bmatrix} = \begin{bmatrix} .75 & .25 \\ .5 & .5 \\ .25 & .75 \end{bmatrix}$

g) f($10 $\rightarrow$ $20) = **.5**

h) $P(T_5=1) = P(5 \rightarrow 0) =$ **.5**;
$P(T_5=2) =$ **0**;
$P(T_5=3) = P(5 \rightarrow 10)\, P(10 \rightarrow 5)\, P(5 \rightarrow 0) + P(5 \rightarrow 10)\, P(10 \rightarrow 15)\, P(15 \rightarrow 20) = 2*.5^3 = 2*.125$
= **.25**

i) U(10, 5) = **1**