

11 Summary & Review

11.1 Art of Hypothesis testing revisited

1. Clarify assumptions and express things in a mathematical model
2. Formulate hypotheses
3. Compute a relevant test statistic
4. Perform test and obtain P-value
5. Draw conclusion

11.2 Checking Normality

Visual inspection (histogram; boxplot; normal-QQ plot);

Shapiro-Wilk test of normality

Example

11.3 Topic-Method Grid

<i>Topic</i>	<i>Data</i>	<i>Procedure</i>	<i>Estimator/CI</i>	<i>Equivalent Parametric Procedures</i>	<i>Note</i>
Checking normality	Any	Visual inspection (histogram; boxplot; normal-QQ plot); Shapiro-Wilk test of normality			hist; boxplot; qqnorm; shapiro.test;
Binomial	Outcome of independent Bernoulli trials	Binomial test	Binomial estimator, CI		rbinom
One-sample location	Paired replicates	Wilcoxon signed rank test	Hodges Lehmann estimator (median of Walsh	t-test, sample mean	wilcox.test

			averages)		
	Paired replicates	Fisher sign test	Sample median	t-test, sample mean	
	One-sample Z_1, \dots, Z_n	Similar	Similar		
Two-sample location	X_1, \dots, X_m and Y_1, \dots, Y_n	Wilcoxon-Mann-Whitney rank sum test	Hodges-Lehmann estimator	Two-sample t-test, difference of sample means	wilcox.test
Two-sample dispersion	X_1, \dots, X_m and Y_1, \dots, Y_n	Ansari-Bradley test		Two-sample F-test	ansari.test
Two-sample difference	X_1, \dots, X_m and Y_1, \dots, Y_n	Kolmogorov-Smirnov			ks.test
One-way layout	X_{ij}	Kruskal-Wallis test		One-way ANOVA	kruskal.test
Two-way layout	X_{ijt}	Friedman test		Two-way ANOVA	friedman.test
Independence	$(X_1, Y_1), \dots, (X_n, Y_n)$	Kendall test Spearman test	Kentall's tau Spearman's rank correlation	Correlation Correlation	cor(...) cor(...)
Comparing two rates	Outcome of two independent sets of Bernoulli trials	Pearson test Fisher exact test			chisq.test fisher.test

11.4 Final notes

People use "nonparametric statistics" to mean different things
Kernel density estimation, nonparameteric regression etc...

Thank you!

11.5 HW #4 Solutions

8.1. With $n=10$ and $\alpha=.054$, $k_{.054}=19$. Test $H_0: \tau=0$ vs $\tau>0$. $K=-7$. Can't reject H_0 since $K<19$.

8.4*. $P(Y_2 > Y_1, X_2 > X_1) = 3/8$ after multiple integration. By symmetry, $P(Y_2 < Y_1, X_2 < X_1) = P(Y_2 > Y_1, X_2 > X_1) = 3/8$. Thus, $\tau = 2(3/8 + 3/8) - 1 = \frac{1}{2}$.

8.20. $\tau\text{-hat} = 2K/(n(n-1)) = 2(-7)/(10 \cdot 9) = -.1556$

10.12 In R, run

```
fisher.test(matrix(c(0,5,2,1),byrow=TRUE,nrow=2), alternative='less')
```

It gives P-value 0.1071. Can't reject the null hypothesis at $\alpha = .05$.

10.13. $E_{11} = 1.25$, $E_{21} = .75$, $E_{12} = 3.75$, $E_{22} = 2.25$ and $\chi^2 = 4.444$. The P-value = $P(\chi^2 > 4.444) = .035$.

We reject the null hypothesis at $\alpha = .05$.

11.5.1 Chapter 7 (Two-way analysis) practice question.

Go over the lecture note and try Problem 7.1.

The solution is below:

"For testing $H_0: \tau_1 = \tau_2 = \tau_3$ vs $H_1: \tau_i$ are not all equal, use Friedman test. Obtain $R_1 = 9.5$, $R_2 = 5$, $R_3 = 9.5$. The statistic $S' = 12[(9.5-8)^2 + \dots] / (12 \times 4 - \frac{1}{2}(22+13-3)) = 3.6$ and the approximate P-value provided by Minitab is $p = P(S' > 3.6) = 0.166$."