Homework #2

In Chapter 3, do #1, 2, 6,7, 16; 18, 27, 36 (the first part only), 37, 41, 42
Do all by hand & simple calculator except: for #16, 42, use R or any other software including 'StatExact'.
#14, 22, 23 (Graduate students only)

Homework #2 Solutions

#1.

```
       x    y    z      R psi
[1,] 350  480  130  130 5   1 5
[2,] 200  130  -70   70 4   0 0
[3,] 240  250   10   10 1   1 1
[4,] 290  310   20   20 2   1 2
[5,]  90  280  190  190 6   1 6
[6,] 370 1450 1080 1080 7   1 7
[7,] 240  280   40   40 3   1 3
```

$T^+ = 24$
When n=7, $P(T^+ >= 24) = .055$.

#2. Changing 1.62 to 16.2 doesn't change the value of T+ and thus P-value remains the same. A change in a single value affects the outcome most severly when it leads to a very big $R_i\psi_i$. This can be obtained by having very large Z. For example, changing from 1.62 to -10 will leads to $R_3\psi_3=9$ and T^+ changes to 14. The P-value then changes to 0.180

#6. 0 and n(n+1)/2, occurring when all Z_is are negative and positive, respectively.

#7. n=14. For H_0: $\theta=0$ vs H1: $\theta>0$ based on T^+,
Exact $\alpha=0.039$ level test rejects when $T^+>=81$.
Test based on large-sample approximation rejects when $(T^+-n*(n+1)/4)/\text{sqrt}(n(n+1)(2n+1)/24) = (T^+-52.5)/ 15.92953 >= z_{0.039} = 1.762$ or equivalently, when $T+ >= 1.762 * 15.92953 + 52.5 = 80.574$. So two critical regions are identical. Thus the .039 nominal level test test based on the large sample approximation has the same exact significance level as the exact test, which is 0.039.

#14. P(Z1+Z2>0) is the proportion of the area of the first (both Zi are positive) and the their (both Zi are negative) quadrants, which is $\frac{1}{2}$ (Think of bivariate normal distribution). In that case, $\eta^* = \frac{1}{2} - \frac{1}{2} = 0$.

#16. P-values are 0.03554 (exact) and 0.03258 (normal approximation) respectively.

```
> wilcox.test(data$x - data$y, alternative = 'greater' )

        Wilcoxon signed rank test with continuity correction

data:  data$x - data$y
V = 62.5, p-value = 0.03554
```

```
alternative hypothesis: true mu is greater than 0

Warning message:
Cannot compute exact p-value with ties in: wilcox.test.default(data$x - data$y,
alternative = "greater")
> wilcox.test(data$x - data$y, alternative = 'greater', exact=FALSE, corr=FALSE)

       Wilcoxon signed rank test

data:  data$x - data$y
V = 62.5, p-value = 0.03258
alternative hypothesis: true mu is greater than 0
```

#18 Walsh averages:

```
       [,1] [,2] [,3] [,4] [,5] [,6] [,7] [,8] [,9] [,10] [,11] [,12]
 [1,] -1100 -900 -750 -700 -400 -175 -150  100  150   250   400   650
 [2,]    NA -700 -550 -500 -200   25   50  300  350   450   600   850
 [3,]    NA   NA -400 -350  -50  175  200  450  500   600   750  1000
 [4,]    NA   NA   NA -300    0  225  250  500  550   650   800  1050
 [5,]    NA   NA   NA   NA  300  525  550  800  850   950  1100  1350
 [6,]    NA   NA   NA   NA   NA  750  775 1025 1075  1175  1325  1575
 [7,]    NA   NA   NA   NA   NA   NA  800 1050 1100  1200  1350  1600
 [8,]    NA   NA   NA   NA   NA   NA   NA 1300 1350  1450  1600  1850
 [9,]    NA   NA   NA   NA   NA   NA   NA   NA 1400  1500  1650  1900
[10,]    NA   NA   NA   NA   NA   NA   NA   NA   NA  1600  1750  2000
[11,]    NA   NA   NA   NA   NA   NA   NA   NA   NA    NA  1900  2150
[12,]    NA   NA   NA   NA   NA   NA   NA   NA   NA    NA    NA  2400

Ordered Walsh averages:
[1] -1100  -900  -700  -750  -550  -400  -700  -500  -350  -300  -400  -200
[13]   -50     0   300  -175    25   175   225   525   750  -150    50   200
[25]   250   550   775   800   100   300   450   500   800  1025  1050  1300
[37]   150   350   500   550   850  1075  1100  1350  1400   250   450   600
[49]   650   950  1175  1200  1450  1500  1600   400   600   750   800  1100
[61]  1325  1350  1600  1650  1750  1900   650   850  1000  1050  1350  1575
[73]  1600  1850  1900  2000  2150  2400
```

Median of them: $\hat{\theta}$=650

#22. Easy to illustrate.

#23. (a) it increases by b. (b) it is multiplied by d. (c) nothing happens.

#27. (-25, 605)

Sorted Ws:

```
[1,]  -70  -30  -25  -15   10
[2,]   15   20   25   30   30
[3,]   40   60   70   75   85
[4,]  100  105  115  130  160
[5,]  190  505  545  550  560
[6,]  605  635 1080
```

For n=7, $\alpha/2$=.023, $t_{\alpha/2}$=26. Cα=28+1-$t_{\alpha/2}$ = 29-26=3. 3rd and 26th ranking Ws are -25 and 605.

* If R is used, the answer becomes (-30, 635), which is more accurate.

#36 Bigger α leads to narrower confidence intervals.

#37 (-30.0, 605.0)

#41 For n=15, α=0.032, t_α=93. The approximation is 93.611

#42 It's (-49.99995, 1350.00001)

```
> wilcox.test(data$x - data$y, conf.int=TRUE, conf.level=0.936)

       Wilcoxon signed rank test with continuity correction

data:  data$x - data$y
V = 62.5, p-value = 0.07108
alternative hypothesis: true mu is not equal to 0
93.6 percent confidence interval:
  -49.99995 1350.00001
sample estimates:
(pseudo)median
           650
```