

Midterm Solution

- 1 (15 pt) Three radar sets A, B and C, operating independently, are set to detect any aircraft flying through a certain area. Each set has a probability of .10 of failing to detect a plane in its area.
- a) If an aircraft enters the area, what's the probability that it goes undetected?
 $.1^3 = .001$
- b) If an aircraft enters the area, what is the probability that it is detected by all three radar sets?
 $.9^3 = .729$
- c) What is the conditional probability that a plane was detected by the radar set A, given that it was detected (by any of A, B or C)?
 $P(A|A \cup B \cup C) = P(A)/(1-.001) = .9/.999 = .9009$
- 2 (30 pt) The telephone lines serving an airline reservation office are all busy about 60% of time. $Y \sim$ Geometric distribution with $p=.4$ success probability
- a) If you're calling this office, what is the probability that you will complete your call on the first try?
 $P(Y=1) = p = .4$
- b) If you're calling this office, what is the probability that it takes more than two calls to complete your call?
 $1-P(Y \leq 2) = 1-p(1)-p(2)=1-.4-.6*.4 = .36$
- c) What are the mean and variance of the number of calls you have to make to complete the call?
 $E(Y) = 1/p = 2.5$ and $\text{var}(Y) = V(Y) = (1-p)/p^2 = (1-.4)/.4^2 = 3.75$
- d) If you have to complete two calls (one for airline and the other for hotel), what are the mean and variance of the number of calls you have to make to complete the two calls?
 $Y' \sim$ Negative Binomial, with $r=2$ and $p=.4$.
 $\mu=E(Y)=r/p = 2/.4 = 5$ and $\sigma^2=V(Y) = r(1-p)/p^2 = 2*(1-.4)/.4^2 = 7.5$
- e) (10 pt) If you and your friend must both complete calls to this office, what is the probability that a total of four tries will be necessary for both of you to get through?
Let $X \sim \text{Geometric}(.4)$ be the random variable for your friend.
 $P(\text{four tries}) = P(X=1, Y=3) + P(X=2, Y=2) + P(X=3, Y=1)$
 $= .4*.6^2*.4 + .6*.4 * .6*.4 + .6^2*.4 * .4 = .1728$

3 (30 pt) A jury of 5 persons was selected from a group of 15 potential jurors, of whom 8 were Asian Americans and 7 were non-Asians Americans. The jury was supposedly randomly selected.

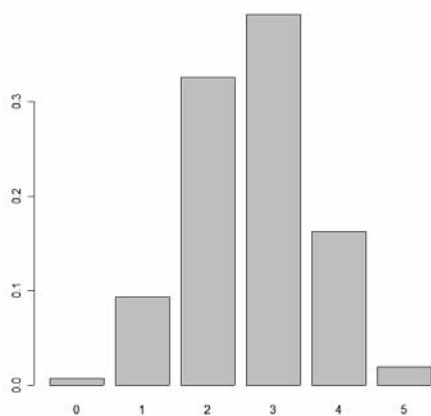
a) Let Y be the number of Asian Americans in the jury. Compute the table of probability function for the distribution of Y.

`round(dhyper(0:5, 8, 7, 5),3)`

`[1] 0.007 0.093 0.326 0.392 0.163 0.019`

b) Draw the probability histogram for Y.

`barplot(round(dhyper(0:5, 8, 7, 5),3), names=0:5)`



c) What is the mean of Y?

brute force: $\sum(y \cdot p(y)) = \sum(0.000 \ 0.093 \ 0.652 \ 1.176 \ 0.652 \ 0.095) = 2.668$.

Using the formula $(nr/N = 5 \cdot 8/15 = 2.667)$ is OK too.

d) What is the probability that there will be no Asian Americans in the jury?

$P(Y=0) = \text{choose}(8,0) \cdot \text{choose}(7,5) / \text{choose}(15,5) = .00699$

e) What is the probability that there will be one or zero Asian American in the jury?

$p(0)+p(1) = .007 + .093 = .1$

f) The final jury contains only 1 Asian American member. Do you have any reason to doubt the randomness of the selection?

There's 10% chance of having one or less Asian Americans in the jury. The conclusion is rather subjective – give them a full credit if their logic makes sense.

4 (20 pt) At a corporation, employees are randomly tested for use of a particular illegal drug. All employees fall into one of the following three mutually exclusive categories: F = frequent user of this drug (1%), R = occasional user (9%), N = never use (90%). The probability of a positive

test for drug use is 90% for group F, 50% for group R and 5% for group N. Answer the following questions.

$$P(F)=.01, P(R)=.09, P(N)=.90$$

$$P(+|F)=.90, P(+|R)=.50, P(+|N)=.05$$

- a) What percentage of all company employees never use the drug and also get a positive test?

$$P(N \& +)=P(+|N)P(N) = .05 \cdot .90 = .045 \text{ or } 4.5\%$$

- b) What percentage of all company employees get a positive test?

$$P(+) = P(N \& +) + P(R \& +) + P(F \& +) = P(+|N)P(N) + P(+|R)P(R) + P(+|F)P(F) \\ = .045 + .045 + .009 = .099$$

- c) Among employees getting a positive test, what proportion never uses the drug?

$$P(N|+) = P(N \& +) / P(+) = .045 / .099 = .4545$$

- d) Among employees getting a negative test, what proportion uses the drug frequently?

$$P(F|-) = P(F \& -) / P(-) = P(-|F)P(F) / (1-P(+)) = .10 \cdot .01 / (1-.099) = .00111$$

- 5 (25 pt) Two guinea pigs mate, producing a litter of 10 offspring. According to the rules of genetics, independently and at random each offspring has probability 1/4 of having straight hair and probability 3/4 of having curly hair.

- a) Let X be the number of guinea pigs in this litter that have straight hair. Name the probability distribution of X, and give its parameter(s).

$$\text{binomial}(10, 1/4)$$

- b) Evaluate $P(X = 2)$.

$$\text{choose}(10,2) \cdot (1/4)^2 \cdot (3/4)^8 = 0.282$$

- c) Evaluate the mean and standard deviation of this distribution.

$$\mu = 10 \cdot 1/4 = 2.5, \text{SD} = \sqrt{10 \cdot 1/4 \cdot (1-1/4)} = 1.37$$

- d) Suppose that male and female guinea pigs are equally likely and that sex is independent of whether hair is curly or straight. What is the probability that a littermate is a female with curly hair?

$$P(\text{female} \& \text{curly}) = P(\text{female}) \cdot P(\text{curly}) = 1/2 \cdot 3/4 = 0.375$$

- e) What is the probability that none of the 10 littermates is a female with curly hair?

$$P(\text{no female} \& \text{curly among four}) = (1-0.375)^{10} = 0.00909$$