Stat 3401/4412 Spring 2005 Jaimie Kwon Name__________________

Quiz #2

- Open book and open note. Use a simple calculator if necessary. Show your work.

1. (6 pts) A parking lot has two entrances. Cars arrive at entrance I according to Poisson distribution at an average rate of three per hour and at entrance II according to a Poisson distribution at an average of four per hour. Assume that the numbers of cars arriving at the two entrances are independent.
   a. What is the probability that a total of two cars will arrive at the parking lot in a given hour?
   b. What is the conditional probability that only one car has arrived at the entrance I in a given hour, given that the total of two cars have arrived at the parking lot in the given hour?

2. (12 pts) The length of time to failure (in hundreds of hours) for a transistor is a random variable Y with distribution function given by $F(y) = \begin{cases} 0, & y < 0 \\ 1 - e^{-y}, & y \geq 0 \end{cases}$

   (Or, F(y)=0 if y is less than 0 and F(y)=1-exp(-y) when y is greater than 0).

   a. Find the density function f(y).
   b. Find the probability that the transistor operates for at least 200 hours.
   c. Find P(Y>1|Y<2).
   d. What are the two names of this probability distribution? Also, find the mean and variance.

3. (6 pts) The percentage of impurities per batch in a cemical product is a random variable Y with density function f(y) = 12 y^3 $(1-y)^2$ (or "12 y cubed times (1-y) squared") over 0<y<1 and zero elsewhere.
   a. What is the probability that a randomly selected batch has more than 50% impurities?
   b. Find the mean and variance of the percentage of impurities in a randomly selected batch.

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Quiz #2 Solutions

1.
   a.
      Let X and Y be the numbers of cars arriving at entrance I and II. i.e., X~Poisson(3), Y~Poisson(4).

      P(X+Y=2) = P(X=0,Y=2) + P(X=1,Y=1) + P(X=2, Y=0)

      = P(X=0)P(Y=2) + P(X=1)P(Y=1) + P(X=2)P(Y=0)

      = exp(-3)*exp(-4)*(3^0 * 4^2/2 + 3^1* 4^1 + 3^2/2 * 4^0) = 0.0223

      Or, use the fact that W=X+Y ~ Poisson(7) and compute P(W=2)
   b. P(X=1|X+Y=2) = P(X=1)P(Y=1) /P(X+Y=2) = dpois(1,4)*dpois(1,3)/dpois(2,7) = 0.4898
2.
   a. f(y)=exp(-y)
   b. P(Y>2)=1-F(2) = exp(-2)=0.1353 (take out -1 for not scaling and compute exp(-200))
   c. P(Y>1|Y<2)=(1-F(1))/F(2) = exp(-1)/(1-exp(-2)) = 0.4255
   d. The probability distribution is Gamma(1,1) or Exp(1). The mean and variance are both 1.

3. It is Beta(4,3) distribution. The formula in the question actually is incorrect since the constant has to be 6!/3!2! = 6*5*4/2 = 60. We will use the corrected f(y) = 5*12 y^3 $(1-y)^2$ below.
   a. f(y)=5(12*y^3*(1-2y+y^2) = 12*y^3 – 24*y^4 + 12*y^5)

      F(y) = 5*(3*y^4 – 24/5*y^5 + 2*y^6)

      P(Y<.5) = F(.5) = .34375, so P(Y>.5) = 1-.34375 = .65625

      * With the incorrect constant, you will get either 1-.34375/5 = .93125 or .65625/5 = .13125. Both answers are correct.

   b. Using the mean and variance formula for beta, $\mu = E(Y) = \dfrac{\alpha}{\alpha+\beta}$ and

      $\sigma^2 = V(Y) = \dfrac{\alpha\beta}{(\alpha+\beta)^2(\alpha+\beta+1)}$, E(Y) = 4/(4+3)=4/7 = .5714 and V(Y) = 4*3/(7^2*8) = .0306.

      * Again, any effort with incorrect constant will get full credit.