3.9 Moments and MGFs

- How do we characterize a distribution? - By p(y). What else?
- Definition 3.12 the "k-th moment of a random variable Y taken about the origin" is defined to be $E(Y^k)$ and is denoted by μ'_k.
- Definition 3.13 The "k-th moment of a random variable taken about its mean", or the k-the "central moment of Y" is defined to be $E[(Y-\mu)^k]$ and is denoted by μ_k.
- In particular, $\mu=\mu'_1$ and $\sigma^2=\mu_2$.
- Under some general conditions, two random variables, Y and Z, that have identical corresponding moments about the origin have identical probability distributions.
- Definition 3.14 The "moment-generating function m(t) for a random variable Y" is defined to be $m(t) = E(e^{tY})$. We say that a MGF for Y exists if there exists a positive constant b such that m(t) is finite for $|t|\le b$.
  - $E(e^{tY})$ is a function of all the moments μ'_k about the origin.
- *Theorem 3.12 If m(t) exists, the for any positive integer k,*
  $$\left.\frac{d^k m(t)}{dt^k}\right]_{t=0} = m^{(k)}(0) = \mu'_k .$$
  - Proof:
- Example 3.23, 24 Find the MGF m(t) of Y~Poisson(λ) distribution. Use it to find the mean and variance of Y.
- *Theorem: If m(t) exists for a probability distribution p(y), it is unique*
- Used to establish the equivalence of two probability distributions
- Example 3.25 If Y is a RV with MGF $m_Y(t)=\exp[3.2(e^t-1)]$, what's the distribution of Y?
- HW. Some of the exercises 3.115~127
- Keywords: MGF

3.10 PGF

3.11 Tchebysheff's theorem

- *Theorem 3.14 Let Y be a random variable with mean μ and finite variance σ^2. Then for any constant k>0,* $P(|Y-\mu|<k\sigma)\ge 1-\frac{1}{k^2}$ *or* $P(|Y-\mu|\ge k\sigma)\le 1-\frac{1}{k^2}$
- It applies to ANY probability distribution and (thus) very "conservative".
- Example. Y~??(20, 2^2). What's P(16<Y<24)=?
- HW. Some of the exercises 3.115~143.