3.8 The Poisson probability distribution

- Question: What's the probability distribution of the number of automobile accidents, Y, at a particular intersection over a week?
- To answer, split the 1-week interval into subintervals so small that:
  P(no accidents occur in a subinterval)=1-p
  P(one accidents occurs in a subinterval)=p
  P(more than one accident occurs in a subinterval)=0
- Also, assume the occurrence of accidents are independent from interval to interval
- Then, Y~bin(n, p) and it makes sense to let λ=np (*why?*)
- *Derivation: Then,* $p(y) = \lim_{n\to\infty} \binom{n}{y} p^y (1-p)^{n-y}$ *=?*
- Written Y~Poisson(λ)
- *Corollary: Poisson(λ) ~ bin(n,p) for large n and small p. (when, say, λ=np < 7)*
- A good model for the number Y of rare events that occur in space, time, volume, or any other dimensions where λ is the average value of Y. Other examples?
- Definition 3.11 A random variable Y is said to have a "Poisson probability distribution" iff

  $p(y) = \frac{e^{\lambda} \lambda^y}{y!}$, y=0,1,2,… λ>0
- Use Table 2 for values of e^{-x} for various x values.
- Example 3.18 Check 0≤p(y)≤1 and $\sum_y p(y)=1$.
- Example 3.19 Y=# of times per half-hour period a patrol visit a given beat location ~ Poisson(1). What's P(Y=0)? What's P(Y=1)? P(Y=2)? P(Y≥1)?
- Example 3.20 Seedlings randomly dispersed in a large area with the mean density 5/yd^2. If a forester randomly locate ten 1-yd^2 sampling regions in the area, P(none of the regions will contain seedlings)=?
- Table 3. Appendix III. Partial sums P(Y≤a) for Poisson distributions.
- Example 3.21 (Poisson approximation to binomial distribution) Y~bin(20, .1). Find the exact and approximate value of P(Y≤3).
- *Theorem 3.11 If Y~Poisson(λ), then μ=E(Y)=λ and σ^2=V(Y)=λ.*
  - Proof:
- Example 3.22 Y = # of industrial accidents at a site per month ~ Poisson(3). If Y=6, does it seem highly improbable if μ=3? Does it indicate an increase in the mean μ?
- HW. Some of the exercises 3.97~114
- Keywords: Poisson distribution