

### 3.4 The binomial probability distribution

- ◆ Definition 3.6 A “binomial experiment” possesses the following properties:
  - A fixed number,  $n$ , of trials
  - Each trial results in one of two outcomes, Success and Failure (or S and F) [dichotomous or binary]
  - Each trial has the same success probability,  $p$  [identical]
  - The trials are independent [independence]
- ◆ Let  $Y$  = # of successes observed during  $n$  trials
- ◆ Example 3.5 and 3.6.
  - $Y_1$  = # of radar units (out of 4) that do not detect the plane
  - $Y_2$  = # of persons favoring a candidate in a random sample of  $n=10$  voters
  - Do they follow binomial distribution?
- ◆ Conditional/unconditional probabilities ;
  - The notion of hypergeometric distribution
- ◆ Definition 3.7 A random variable  $Y$  is said to have a “binomial distribution” based on  $n$  trials with success probability  $p$  if and only if
  - $p(y) = \binom{n}{y} p^y q^{n-y}$ ,  $y=0,1,\dots,n$  and  $0 \leq p \leq 1$
  - Histograms? (Use R)
- ◆ Example 3.7 5000 fuses contain 5% defectives; for a sample of 5 fuses, what's  $P(Y \geq 1)$ ?
  - How about  $P(Y \geq 4)$ ?
- ◆ Notation:  $Y \sim \text{bin}(n,p)$
- ◆ Theorem 3.7 Let  $Y \sim \text{bin}(n,p)$ . Then,
  - $\mu = E(Y) = np$  and  $\sigma^2 = V(Y) = npq$
  - Proof: definitions and ‘binomial’ trick
- ◆ In practical application, carefully define ‘successes!’
- ◆ HW. Some of the exercises 3.25~49
- ◆ Keywords: binomial experiment;  $Y \sim \text{bin}(n,p)$