

2.8 Two laws of probability

- ◆ *Theorem 2.5. (The multiplicative Law of Probability) For two events A and B,*

$$P(A \cap B) = P(A)P(B|A) = P(B)P(A|B).$$

If A and B are independent, then

$$P(A \cap B) = P(A)P(B)$$

- Proof: follows from the definitions
- Applying it multiple times, we can have results like

$$P(A_1 \cap A_2 \cap \dots \cap A_k) =$$

$$P(A_1)P(A_2 | A_1)P(A_3 | A_1 \cap A_2) \dots P(A_k | A_1 \cap A_2 \cap \dots \cap A_{k-1})$$

- ◆ *Theorem 2.6. (The additive Law of Probability) For two events A and B,*

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

If A and B are mutually exclusive, (so $P(A \cap B) = 0$) then

$$P(A \cup B) = P(A) + P(B)$$

- Proof: inspection of the Venn diagram
- $P(A \cup B \cup C) = ?$

- ◆ *Theorem 2.7. $P(A) = 1 - P(A')$*

- Proof: use the previous theorem.

- ◆ **HW: some of the exercises 2.66~85**

- ◆ **Keywords: the multiplicative & additive law of probability;**

2.9 Calculating the probability of an event: the event-composition method

- ◆ Example 2.17. Political composition of a city and their support of the bond issues

R vs. D (40% vs. 60%), $P(F|R) = .7$ and $P(F|D) = .8$. What is $P(F)$?

- ◆ Example 2.18. Birthday problem. We know $P(A) = .5886$. What's $P(B) = P(A')$?

- ◆ *Steps for the event-composition method (use the multiplicative/additive laws)*

- Define the experiment
- Visualize the nature of the sample points. Identify a few to clarify your thinking.
- Write an equation expressing the event of interest, say A, as a composition of two or more events, using unions, intersections and/or complements. (*hardest*)
- Apply the laws of probability to the compositions to find $P(A)$

- ◆ Example 2.19. (same as example 2.12) Selecting two applicants randomly from five. $P(\text{exactly one of the two best applicants are selected})=?$

.. $=P(\text{draw the best and one of the three poorest}) + P(\text{draw the second best and one of the three poorest})$

$= P(\text{draw the best on the first draw})P(\text{draw one of the three poorest on the second draw}) + \dots$

- ◆ Example 2.20. $P(\text{a patient with a disease respond to a treatment})=.9$.
 $P(\text{at least one of three patients, treated independently, will respond})=?$
* It's easier to find the probability of the complement

- ◆ Example 2.21. $P(\text{a new arrival at a medical clinic is an emergency case})=1/6$. $P(\text{r'th patient is the first emergency case})=?$
Also check $P(S) = 1$

- ◆ Example 2.22 (Monkey color recognition experiment)

- ◆ HW. Some of the exercises 2.86~97
- ◆ Keywords: the event-composition method