2.7 Conditional probability and the independence of events

- Illustration: The probability of rain on a given day ignoring atmospheric conditions or any other events is the fraction of days in which rain occurs over a long period of time – the "unconditional probability". But given it has rained past two days and a storm is approaching, what's the probability of rain tomorrow? – the "conditional probability"
- Definition 2.9. The "conditional probability of an event A given an event B" or "the probability of A given B" is $P(A|B)=\frac{P(A\cap B)}{P(B)}$, provided P(B)>0.
- Consistency of this definition with the relative frequency concept of probability can be illustrated by enumerating P(A), P(B), P(A|B), P(B|A), P(A∩B) in the following construction:

	A	A'	
B	n_{11}	n_{12}	$n_{11}+n_{12}$
B'	n_{21}	n_{22}	$n_{21}+n_{22}$
	$n_{11}+n_{21}$	$n_{12}+n_{22}$	N

Table for events A and B

- Example 2.14. P(1|odd) for balanced die tossing
- If the occurrence of an event A is not affected by (non-)occurrence of event B, than the two events are independent. More formally,
- Definition 2.10. Two events A and B are "independent" if any one of the following holds:
  P(A|B)=P(A)
  P(B|A)=P(B), or
  P(A∩B)=P(A)P(B).
  Otherwise, the two events are said to be "dependent".
- Illustrations: smoking and lung cancer. Rain today and rain a month from now.
- Example 2.15. A single die-tossing. Let
  A: observe an odd number
  B: observe an even number
  C: Observe a 1 or 2
  Are A and B independent? Are A and C independent?
- Example 2.16. Coffee ranking
- HW: some of the exercises 2.57~65
- Keywords: conditional prabability; independence