2.7 **Conditional probability and the independence of events**

- Illustration: The probability of rain on a given day ignoring atmospheric conditions or any other events is the fraction of days in which rain occurs over a long period of time – the “unconditional probability”. But given it has rained past two days and a storm is approaching, what’s the probability of rain tomorrow? – the “conditional probability”

- Definition 2.9. The “conditional probability of an event A given an event B” or “the probability of A given B” is

\[
P(A | B) = \frac{P(A \cap B)}{P(B)}, \text{ provided } P(B) > 0.
\]

- Consistency of this definition with the relative frequency concept of probability can be illustrated by enumerating \(P(A), P(B), P(A|B), P(B|A), P(A \cap B)\) in the following construction:

<table>
<thead>
<tr>
<th></th>
<th>A</th>
<th>A’</th>
</tr>
</thead>
<tbody>
<tr>
<td>B</td>
<td>(n_{11})</td>
<td>(n_{12})</td>
</tr>
<tr>
<td>B’</td>
<td>(n_{21})</td>
<td>(n_{22})</td>
</tr>
<tr>
<td></td>
<td>(n_{11} + n_{21})</td>
<td>(n_{12} + n_{22})</td>
</tr>
</tbody>
</table>

Table for events A and B

- Example 2.14. \(P(1|\text{odd})\) for balanced die tossing

- If the occurrence of an event A is not affected by (non-)occurrence of event B, than the two events are independent. More formally,

- Definition 2.10. Two events A and B are “independent” if any one of the following holds:

\[
P(A|B) = P(A),
\]

\[
P(B|A) = P(B), \text{ or}
\]

\[
P(A \cap B) = P(A)P(B).
\]

Otherwise, the two events are said to be “dependent”.

- Illustrations: smoking and lung cancer. Rain today and rain a month from now.

- Example 2.15. A single die-tossing. Let

A: observe an odd number

B: observe an even number

C: Observe a 1 or 2

Are A and B independent? Are A and C independent?

- Example 2.16. Coffee ranking

- HW: some of the exercises 2.57–65

- Keywords: conditional probability; independence