

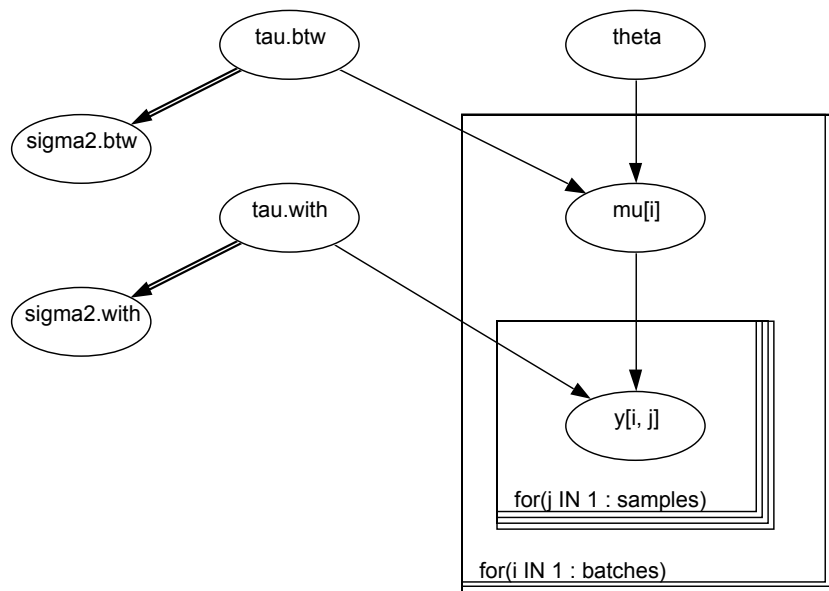
Illustration of the model:

$$Y_{ij} = \mu + A_i + e_{ij},$$

$$y_{ij} \sim \text{Normal}(\mu_i, \tau_{\text{within}})$$

$$\mu_i \sim \text{Normal}(\theta, \tau_{\text{between}})$$

where y_{ij} is the yield for sample j of batch i , μ_i is the true yield for batch i , τ_{within} is the inverse of the within-batch variance σ_{within}^2 (i.e. the variation due to sampling and analytic error), θ is the true average yield for all batches and τ_{between} is the inverse of the between-batch variance $\sigma_{\text{between}}^2$. The total variation in product yield is thus $\sigma_{\text{total}}^2 = \sigma_{\text{within}}^2 + \sigma_{\text{between}}^2$ and the relative contributions of each component to the total variance are $f_{\text{within}} = \sigma_{\text{within}}^2 / \sigma_{\text{total}}^2$ and $f_{\text{between}} = \sigma_{\text{between}}^2 / \sigma_{\text{total}}^2$. We assume standard non-informative priors for θ , τ_{within} and τ_{between} .

Graphical model

WINGBUGS Code:

```
# Classroom Simulation: Understanding One-Way Random-Effect ANOVA
# Eric A. Suess, Bruce E. Trumbo, and Yun Jiang, CSU, East Bay
# Poster: Section on Statistics Education, JSM 2005, Minneapolis, MN
```

```
# Fig. 9. WinBUGS Code for Second Parameterization, dataset1
# Uniform prior for ICC
```

```
model
{
  for( i in 1 : batches ) {
    mu[i] ~ dnorm(theta, tau.btw)
    for( j in 1 : samples ) {
      y[i , j] ~ dnorm(mu[i], tau.with)
    }
  }
  theta ~ dnorm(0.0, 1.0E-10)

  # prior for within-variation
  sigma2.with <- 1 / tau.with
  tau.with ~ dgamma(0.001, 0.001)
  sigma.with <- sqrt(sigma2.with)
  sigma.btw <- sqrt(sigma2.btw)

  # Prior for ICC
  ICC ~ dunif(0,1)
  sigma2.btw <- sigma2.with *ICC/(1-ICC)
  tau.btw<-1/sigma2.btw
}
```

WINGBUGS language:

DATA

```
# dataset 1
list(batches = 30, samples = 5,
     y = structure(
       .Data = c(959, 976, 1015, 1003, 971,
                 977, 990, 972, 977, 1004,
                 987, 988, 1008, 973, 963,
                 974, 1004, 995, 991, 977,
                 1011, 1009, 984, 1033, 1014,
                 1058, 1063, 1057, 1062, 1072,
```

```

972, 965, 968, 973, 974,
1028, 1047, 1044, 1039, 1047,
973, 1007, 998, 1030, 1001,
1042, 991, 1016, 1014, 1018,
1013, 989, 1016, 992, 1010,
991, 1001, 1024, 977, 999,
978, 939, 960, 990, 990,
1026, 1032, 1017, 1018, 998,
1030, 1024, 1054, 1046, 1061,
1038, 1024, 1005, 990, 1018,
1020, 997, 1019, 1008, 1016,
1039, 1024, 1053, 1038, 1019,
1023, 1009, 1037, 993, 1007,
1006, 994, 1002, 1011, 1007,
993, 1003, 979, 990, 983,
1011, 975, 988, 1017, 999,
982, 1006, 1003, 975, 991,
970, 935, 951, 960, 972,
1008, 987, 977, 981, 1004,
994, 980, 1016, 1023, 1009,
971, 969, 973, 964, 951,
1070, 1050, 1041, 1055, 1047,
985, 995, 998, 995, 989,
1000, 987, 979, 1013, 1008),
.Dim = c(30, 5))

```

Inits

```
list(theta=1500, tau.with=1, ICC=0.5)
```

Results

A 25000 update burn in followed by a further 100000 updates gave the parameter estimates

node	mean	sd	MC error	2.50%	median	97.50%	start	sample
ICC	0.7272	0.06053	3.06E-04	0.6005	0.7305	0.8357	25001	75000
mu.grand	1003	1.2	0.004599	1001	1003	1006	25001	75000
sigma.btw	24.38	3.487	0.01501	18.56	24.03	32.17	25001	75000
sigma.with	14.65	0.9576	0.004843	12.93	14.6	16.68	25001	75000

WINGBUGS Code:

```
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```

```
# Gamma prior for tau.btw =  $1/\sigma_A^2$ , dataset1
# not shown in the paper
```

```
model
{
  for( i in 1 : batches ) {
    mu[i] ~ dnorm(theta, tau.btw)
    for( j in 1 : samples ) {
      y[i , j] ~ dnorm(mu[i], tau.with)
    }
  }
  theta ~ dnorm(0.0, 1.0E-10)

  # prior for within-variation
  sigma2.with <- 1 / tau.with
  tau.with ~ dgamma(0.001, 0.001)

  sigma.with <- sqrt(sigma2.with)
  sigma.btw <- sqrt(sigma2.btw)

  ICC <- sigma2.btw / (sigma2.btw+sigma2.with)

  # Gamma Prior for tau.btw
  tau.btw ~ dgamma(0.001, 0.001)
  sigma2.btw <- 1 / tau.btw
}
```

DATA

```
# dataset 1
list(batches = 30, samples = 5,
     y = structure(
       .Data = c(959, 976, 1015, 1003, 971,
                 977, 990, 972, 977, 1004,
                 987, 988, 1008, 973, 963,
                 974, 1004, 995, 991, 977,
                 1011, 1009, 984, 1033, 1014,
                 1058, 1063, 1057, 1062, 1072,
```

```

972, 965, 968, 973, 974,
1028, 1047, 1044, 1039, 1047,
973, 1007, 998, 1030, 1001,
1042, 991, 1016, 1014, 1018,
1013, 989, 1016, 992, 1010,
991, 1001, 1024, 977, 999,
978, 939, 960, 990, 990,
1026, 1032, 1017, 1018, 998,
1030, 1024, 1054, 1046, 1061,
1038, 1024, 1005, 990, 1018,
1020, 997, 1019, 1008, 1016,
1039, 1024, 1053, 1038, 1019,
1023, 1009, 1037, 993, 1007,
1006, 994, 1002, 1011, 1007,
993, 1003, 979, 990, 983,
1011, 975, 988, 1017, 999,
982, 1006, 1003, 975, 991,
970, 935, 951, 960, 972,
1008, 987, 977, 981, 1004,
994, 980, 1016, 1023, 1009,
971, 969, 973, 964, 951,
1070, 1050, 1041, 1055, 1047,
985, 995, 998, 995, 989,
1000, 987, 979, 1013, 1008),
.Dim = c(30, 5))

```

Inits

```
list(theta=1500, tau.with=1, tau.btw=1)
```

Results

A 25000 update burn in followed by a further 100000 updates gave the parameter estimates

node	mean	sd	MC error	2.50%	median	97.50%	start	sample
ICC	0.7358	0.06078	2.75E-04	0.6068	0.7392	0.8444	25001	75000
mu.grand	1003	1.197	0.004925	1001	1003	1006	25001	75000
sigma.btw	24.86	3.637	0.01624	18.84	24.48	33.02	25001	75000
sigma.with	14.6	0.9521	0.004387	12.88	14.55	16.61	25001	75000

WINGBUGS Code:

```
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```

```
# WinBUGS Code for Second Parameterization, dataset2
```

```
# Uniform prior for ICC
```

```
# not shown in the paper
```

```
model
{
  for( i in 1 : batches ) {
    mu[i] ~ dnorm(theta, tau.btw)
    for( j in 1 : samples ) {
      y[i , j] ~ dnorm(mu[i], tau.with)
    }
  }
  theta ~ dnorm(0.0, 1.0E-10)

  # prior for within-variation
  sigma2.with <- 1 / tau.with
  tau.with ~ dgamma(0.001, 0.001)
  sigma.with <- sqrt(sigma2.with)
  sigma.btw <- sqrt(sigma2.btw)

  # Prior for ICC
  ICC ~ dunif(0,1)
  sigma2.btw <- sigma2.with *ICC/(1-ICC)
  tau.btw<-1/sigma2.btw
}
```

WINGBUGS language:

DATA

```
# dataset 2
```

```
list(batches = 30, samples = 5,
     y = structure(
       .Data = c(1038, 975, 976, 949, 992,
                 984, 997, 1010, 980, 967,
                 1000, 995, 982, 1029, 1008,
                 993, 1006, 1051, 1026, 993,
                 992, 994, 1002, 1003, 1008,
```

```

1052, 987, 974, 991, 989,
988, 1020, 975, 1003, 971,
961, 993, 1012, 976, 1005,
988, 1000, 998, 1012, 1051,
1017, 1012, 966, 993, 1007,
1025, 1022, 1005, 1021, 1022,
948, 1026, 1031, 989, 1008,
995, 997, 984, 968, 990,
996, 1001, 969, 995, 1030,
1022, 996, 1028, 986, 976,
976, 1023, 1004, 1026, 992,
983, 994, 975, 1049, 1002,
988, 1004, 1022, 996, 1011,
1037, 1001, 986, 986, 1018,
965, 994, 997, 1005, 1029,
1017, 1010, 964, 967, 1035,
1053, 954, 997, 1019, 1008,
1028, 992, 1026, 1008, 1020,
1012, 1026, 981, 1034, 1035,
976, 952, 999, 976, 958,
1016, 992, 987, 1010, 1024,
949, 1007, 1009, 958, 1031,
1004, 984, 986, 975, 1039,
1020, 984, 990, 986, 991,
1021, 981, 1006, 1043, 981),
.Dim = c(30, 5))

```

Inits

```
list(theta=1500, tau.with=1, ICC=0.5)
```

Results

A 25000 update burn in followed by a further 100000 updates gave the parameter estimates

node	mean	sd	MC error	2.50%	median	97.50%	start	sample
ICC	0.05654	0.04682	8.66E-04	0.001874	0.04514	0.1768	25001	75000
mu.grand	1.00E+03	1.897	0.02648	996.3	1.00E+03	1004	25001	75000
sigma.btw	5.134	2.415	0.05238	1.008	4.974	10.37	25001	75000
sigma.with	22.88	1.371	0.008108	20.38	22.81	25.74	25001	75000

WINGBUGS Code:

```
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```

```
# Gamma prior for tau.btw =  $1/\sigma_A^2$ , dataset2
# not shown in the paper
```

```
model
{
  for( i in 1 : batches ) {
    mu[i] ~ dnorm(theta, tau.btw)
    for( j in 1 : samples ) {
      y[i , j] ~ dnorm(mu[i], tau.with)
    }
  }
  theta ~ dnorm(0.0, 1.0E-10)

  # prior for within-variation
  sigma2.with <- 1 / tau.with
  tau.with ~ dgamma(0.001, 0.001)

  sigma.with <- sqrt(sigma2.with)
  sigma.btw <- sqrt(sigma2.btw)

  ICC <- sigma2.btw / (sigma2.btw+sigma2.with)

  # Gamma Prior for tau.btw
  tau.btw ~ dgamma(0.001, 0.001)
  sigma2.btw <- 1 / tau.btw
}
```

DATA

```
# dataset 2
list(batches = 30, samples = 5,
     y = structure(
       .Data = c(1038, 975, 976, 949, 992,
                 984, 997, 1010, 980, 967,
                 1000, 995, 982, 1029, 1008,
                 993, 1006, 1051, 1026, 993,
                 992, 994, 1002, 1003, 1008,
                 1052, 987, 974, 991, 989,
```

```

988, 1020, 975, 1003, 971,
961, 993, 1012, 976, 1005,
988, 1000, 998, 1012, 1051,
1017, 1012, 966, 993, 1007,
1025, 1022, 1005, 1021, 1022,
948, 1026, 1031, 989, 1008,
995, 997, 984, 968, 990,
996, 1001, 969, 995, 1030,
1022, 996, 1028, 986, 976,
976, 1023, 1004, 1026, 992,
983, 994, 975, 1049, 1002,
988, 1004, 1022, 996, 1011,
1037, 1001, 986, 986, 1018,
965, 994, 997, 1005, 1029,
1017, 1010, 964, 967, 1035,
1053, 954, 997, 1019, 1008,
1028, 992, 1026, 1008, 1020,
1012, 1026, 981, 1034, 1035,
976, 952, 999, 976, 958,
1016, 992, 987, 1010, 1024,
949, 1007, 1009, 958, 1031,
1004, 984, 986, 975, 1039,
1020, 984, 990, 986, 991,
1021, 981, 1006, 1043, 981),
.Dim = c(30, 5))

```

Inits

```
list(theta=1500, tau.with=1, tau.btw=1)
```

Results

A 25000 update burn in followed by a further 100000 updates gave the parameter estimates

node	mean	sd	MC error	2.50%	median	97.50%	start	sample
ICC	0.008207	0.02026	5.95E-04	2.05E-06	5.21E-04	0.07001	25001	75000
mu.grand	999.9	2.137	0.1014	995.6	999.9	1004	25001	75000
sigma.btw	1.28	1.716	0.06606	0.03355	0.5322	6.279	25001	75000
sigma.with	23.3	1.372	0.008896	20.79	23.23	26.17	25001	75000