Chapter 4 and Parts of Chapter 6: Suggested Minimal Homework Problems

NOTE: Technical details of the continuous distributions in Chapter 4 are especially important background information for Statistics MS students. In particular, mastery of this material is crucial in Stat 6401, 6501, 6502. Knowledge of the exponential and gamma distributions is also important in Stat 4401. Generating functions and Chebyshev’s inequality will be used in Stat 3402. (Tchebysheff is the standard French transliteration of Cyrillic Чебышёв; one can only wonder why it is used in a book written in English.) Stat MS students need to do the extra problems noted.

Chapter 4

Section 2: (Corrected/Expanded 10/29/04)
4.1, 4.2, 4.3, 4.4 [k = 6; (d) = (e) = 0.393],

```r
> x <- c(0, .4)
> diff(6*(x^2/2 - x^3/3))
[1] 0.352
> x <- c(0, .8)
> diff(6*(x^2/2 - x^3/3))
[1] 0.896
> .352/.896
[1] 0.3928571
```
4.5, 4.8 \[ F(y) = 1 - b/y, y \geq b; \ 0 \ otherwise. \], 4.9,
4.11 [(b) \[ F(y) = y^3/2 + y^2/2, for 0 < y < 1; (f) \[ 1 - F(1/2)]/[1 - F(1/4)].]\]

Section 3: 4.14 \[ E(Y^2) = 2/3, V(Y) = 2/9 \], 4.15, 4.17, 4.18, 4.21, 4.27 [Chebyshev].

Section 4: 4.28, 4.31, 4.37, 4.41, 4.44 [(1/6)/(2/3) = 1/4].
4.44 in R:

```r
Num <- punif(30, 0, 30) - punif(25, 0, 30)
Den <- 1 - punif(10, 0, 30)
Ans <- Num/Den
Ans
[1] 0.25
```

Section 5: 4.46 [Draw sketches! .3849, .3158, .3227, .1586, .3613], 4.47, 4.49,
4.52 [.0730], 4.53, 4.59 [R: \[ qnorm(.8531, 950, 10) \] returns 960.4982.], 4.61, 4.65,
4.66 [Hint: \[ E(Y^2) = \sigma^2 + \mu^2 \].]

Section 6: 4.67, 4.68, 4.69, 4.73, 4.77, 4.81, 4.84 [Hint: \( \alpha = \beta = 2 \)],
4.85 [Chebyshev gives the interval 4 \pm 5.6569 which really amounts to (0, 5.6569)]
4.85 in R: The actual probability in the Chebyshev interval is much larger than 3/4 and the interval (0, 5.385) has very nearly probability 3/4:

```r
> pgamma(9.6569, 2, scale=2)
[1] 0.9533788
> qgamma(.75, 2, scale=2)
[1] 5.385269
```

Note: In today's computational age, Chebyshev's inequality should seldom be used in practice to approximate probabilities, but it remains a crucial theoretical tool.

STAT MS: In the gamma family of distributions, the parameter \( \alpha > 0 \) is called the shape parameter for good reason. Consider the behavior of the density curve near 0: Does it approach infinity, approach a nonzero constant, approach 0 with infinite slope, approach 0 with nonzero slope, or approach 0 with 0 slope? How many inflection points does the density curve have? Find five ranges for \( \alpha \) that correspond to five fundamentally different shapes of the density function.

Section 7: 4.91, 4.92 [R: \( 1 - pbeta(.4, 3, 2) \) returns 0.8208], 4.93, 4.95, 4.99.
4.93 in R: Here is an approximation of the mean and variance of BETA(3, 2).
When the parameters are not small integers, analytic methods for finding probabilities and expectations related to beta distributions can be difficult. In some cases, even computational methods have to be done with care to avoid overflow or underflow. (In 4.93 and 4.94, if \( \alpha = 2.99 \) and \( \beta = 1.99 \), you would expect answers not to change much, but straightforward calculus is no longer an option.)

**STAT MS:** Also 4.98. In the beta family of distributions, the parameter \( \alpha \) controls the shape of the density curve near 0 and \( \beta \) controls the shape near 1. Analysis such as the one suggested above for the gamma family shows that there are 25 fundamentally different beta shapes. For what parameter values does a beta density have a mode (absolute maximum) within the interval \((0, 1)\)? For what parameter values does a beta density have no inflection points? one? two?

Section 9: 4.104 through 4.110.  [Ans for 4.108: (a) GAMMA(2, 4), (b) EXP(3.2), (c) NORM(–5, 12).]

Section 10: 4.115 as stated (with no particular distribution stated); also give a precise answer if the fill weights are normally distributed. 4.117, 4.72 (page 180) together with 4.119, 4.122 [(a) 7, 14; (b) 3.742 (c) 23 is 4.276\( \sigma \) above \( \mu \), so by Chebyshev such values are unlikely, having probability less than \( 1/4.276^2 = 0.0546921 \); R: \( 1 - pchisq(23, 7) \) returns 0.001704608. What bound does Markov's inequality give?]

**STAT MS:** Know how to prove Chebyshev's inequality via Markov's inequality as well as using the method in the book. Compare with Chebyshev's inequality for discrete distributions in Chapter 3.

Section 11. No problems, but Stat MS students should read this section and compare with class discussion.

**Studying for the Second Midterm Exam**

Work selected Supplementary Exercises in a haphazard order with emphasis on how to recognize distributions and methods when they appear out of context. Review topics (including computational topics using R) included in lecture but not in the book.

**Chapter 6**

(Consider only univariate transformations. Bivariate transformations require Math 2304: Calc III, and are covered in Stat 3402.)

Examples 6.1 and 6.4
Problem 6.1 [(a) pdf of \( U_1 \) is \( (1 – u)/2 \), for \(-1 < u < 1\), (c) pdf of \( U_2 \) is \( u^{-1/2} – 1 \), for \( 0 < u < 1 \)]
Examples 6.6 and 6.7
Problem 6.19 [(a) \((1 + u)/2\), for \(-1 < u < 1\), (c) \((u^{1/2} – u)/u\), for \( 0 < u < 1 \)]

Also problems in lectures/handouts on both Chapter 5 and Chapter 6.
Use the material covered in class as your main guide to what you need to cover in these chapters.